Deliberation and Security Design in Bankruptcy¹

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Abstract

We consider a bargaining game among the claimants of a bankrupt firm in which claimants have private information about various operational restructuring alternatives, and can communicate prior to an offer. Our setup differs from typical bargaining games with incomplete information in two ways. First, the proposals can be made using securities. Second, the negotiations are over two interdependent issues: what to do with the firm and who gets what. In line with Chapter 11 bankruptcy proceedings we first analyze the case in which both issues are negotiated simultaneously. We show that simultaneous negotiation leads to efficient operational restructuring. Moreover, any efficient equilibrium requires that the original senior claimants receive senior securities of the reorganized firm. Next, we analyze the cases in which the two issues are negotiated sequentially. If the first issue is what to do with the firm, then there is no efficient equilibrium. In contrast, if the first issue is who gets what, then an efficient equilibrium exists. In comparison to simultaneous negotiation, efficient sequential negotiation may result in junior claimant capturing a larger surplus.
1 Introduction

When a firm in the U.S. files for Chapter 11 bankruptcy, negotiations take place among the claimants of the firm to decide what to do with the firm, and how to split the value. The underlying premise for these negotiations is that although each claimant may have some information with respect to the relative desirability of various operational restructuring alternatives, no single claimant has the full information that would enable him to make the optimal decision. U.S. bankruptcy law also encourages the claimants to share their information through direct communication.\(^1\) Despite the availability of the means for information sharing, conflicts of interests may prevent the claimants to reveal their information truthfully. In particular, it is well understood by both the practitioners and the academic literature that junior claimants may have an incentive to reorganize a non-viable firm since they don’t face the downside risk; whereas senior creditors are inclined towards liquidation as they have limited upside potential. This paper analyzes the conditions under which Chapter 11 can resolve conflicts of interest and aggregate dispersed information in order to maximize the firm value. In addition, we characterize the properties of efficient equilibrium outcomes and evaluate the impact of policy changes on efficiency, operational decisions and recoveries of the claimants.

The model we analyze is a multiple issue incomplete information bargaining game with interdependent valuations.\(^2\) There are two claimants of a bankrupt firm each of whom observes a noisy signal that is informative about the firm value under different operational restructuring alternatives. Claimants can communicate their private infor-

\(^1\)Bankruptcy Rule 2003(a) provides that in a Chapter 11 reorganization case, the United States trustee must call a meeting of creditors to be held no fewer than 20 and no more than 40 days after the order of relief.

\(^2\)Despite the large theoretical literature on bargaining, there are few papers that study bargaining with interdependent values. Exceptions are Evans (1989), Vincent (1989), Bond and Eraslan (2006), Deneckre and Liang (2006).
mation through cheap talk that takes place prior to proposal stage.\textsuperscript{3} If an agreement cannot be reached, then the firm is liquidated.

Our key modeling innovations in bargaining theory are twofold. First, proposals can involve securities. This is motivated by the fact that in real life bankruptcy negotiations proposals involve securities such as debt, equity, warrants in addition to cash payments. Second, the negotiations are over two interdependent issues: what to do with the firm (operational restructuring) and who gets what (financial restructuring). To provide an analog to the paradigm of the “divide-the-dollar” game in single-issue environments, we may think of our environment as a “divide-a-currency” game where the dollar value of the surplus to be allocated is random (due to uncertainty of the exchange rate) with a distribution that depends on the currency alternative chosen. In this game agents bargain not only over how to divide a currency, but also which currency to divide to begin with. For this environment we are interested in answering the following questions:

- Does security design make a difference?
- Does the sequencing of the issues to be negotiated matter?

In addition to being of theoretical interest, the answers to these questions shed light on the proposals to reform current U.S. bankruptcy law. Critics argue that Chapter 11 is flawed because it combines the decision of what to with the firm with the decision of how to split the surplus and propose alternative restructuring mechanisms (see, for example, Bebchuck (1988), Aghion, Hart and Moore (1992)). The most important component of these proposals is the separation of the two decisions so that operational restructuring is decided by the claimants while the financial restructuring is decided by

\textsuperscript{3}Again, despite the large literature on bargaining theory, there are few papers that incorporate communication in bargaining. Exceptions are Farrell and Gibbons (1989), Crawford (1990), Chakraborty and Harbaugh (2003) and Meirowitz (2006).
the bankruptcy court. Despite having important implications for bankruptcy policy, there is no research on bargaining over multiple interdependent issues.\textsuperscript{4} We aim to fill this gap in the context of a stylized model of bargaining that captures the key elements of the negotiations in Chapter 11 bankruptcy.

In our model, there are three ways an inefficiency can arise if the conflicts of interests among claimants are significant. First, one or both of the claimants might have an incentive to misrepresent his private information in order to manipulate the decisions of the other. Second, the optimal operational restructuring alternative may not be proposed. Third, even when the optimal restructuring alternative is proposed, it may be voted down. All these sources of inefficiency may be present in both simultaneous and sequential negotiations.

We first analyze the benchmark case that corresponds to the arrangement in Chapter 11 in that the two issues are negotiated simultaneously. Our analysis yields the following results. If financial restructuring is not allowed, then the claimants cannot credibly reveal their information. For example, senior claimants will always report messages that are biased towards liquidation because they do not gain from the upside potential but they bear the downside risk. The lack of meaningful communication leads to inefficient liquidation or operational restructuring decisions. In contrast, when financial restructuring is allowed, there exists efficient equilibria. Moreover, security design plays a crucial role. In particular, all efficient equilibria require issuance of multiple securities and allocation of all of the newly issued senior security to senior claimants. This completely eliminates the claimants’ incentive to misrepresent information. Such a financial restructuring is also consistent with the empirical evidence (see, for example, Franks and Torous (1989)).

\textsuperscript{4}Existing studies on multi-issue bargaining focus on the case where each issue involves a division of a different dollar, and the interaction between the issues are imposed exogenously through utility functions of the players. See, for example, Fershtman (1990, 2000).
Given the criticisms of Chapter 11 mentioned above, a natural question to ask is to what extent the sequencing of the issues matters in obtaining efficient outcomes. To answer this question, we next analyze the effect of a policy change which involves the two issues being negotiated sequentially. As before, efficient equilibrium does not exist if either communication or financial restructuring is not allowed. However, allowing both communication and security design is not sufficient for efficiency. In particular, the order in which the issues are negotiated matters. If the first issue involves operational restructuring decision, then there is never an efficient equilibrium. This is because, once the claimants agree on an operational restructuring alternative, there is no incentive to transfer wealth through financial restructuring. Anticipating the lack of financial restructuring, the claimants do not have the incentive to share information in the first stage. But then, it is not possible to have an efficient equilibrium. In contrast, if the first issue involves the financial restructuring decision, then an efficient equilibrium exists. By choosing a financial restructuring that maximizes each claimant’s payoffs when the optimal operational restructuring is implemented, it is possible to induce information sharing. Our results imply that simultaneous negotiation on two interdependent issues in and of itself does not cause ex post inefficiency. In contrast, sequential negotiation of the two issues may result in ex post inefficiency. In addition to its efficiency implications, the order of negotiations has distributional consequences as well.

Our result on efficiency also makes a contribution to the mechanism design literature on bilateral bargaining. In their seminal work, Myerson and Satterthwaite (1983) show that in a bilateral bargaining problem with independent private values, there exists no ex post efficient mechanism without outside subsidies. Cramton, Bu-

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Footnote:

Note that our analysis exclusively focuses on ex post efficiency. Policy proposals by Bebchuck (1988) and Aghion, Hart and Moore (1992) are mainly aimed at attaining ex ante efficiency by designing a financial restructuring in a way that preserves the ex ante entitlements against the firm.
low, Klemperer (1987) generalize this bilateral bargaining problem to the case where there are many agents, each of whom is endowed with a fraction of an asset that may be traded among them. They show that it is possible to achieve ex post efficient trade provided that no agent owns too large a share. Our setup differs in that the claimants have interdependent values. In this case, it is much harder to achieve efficient trade since the information revealed ex post is always bad news due to winner’s or loser’s curse reminiscent of the no-trade theorems of Grossman and Stiglitz (1980) and Milgrom and Stokey (1982) in pure common values case. Fieseler, Kittsteiner and Moldovanu (2000) show that even if initial shares are equal, it is not always possible to dissolve a partnership efficiently when there is an arbitrarily small common values in valuations. In our model, first best is an equilibrium outcome despite the presence of interdependent payoffs, inherent conflicts of interests, and the absence of commitment. This is because it is possible to propose payments in securities which enables the claimants to credibly reveal their information in the first place.

This paper also makes a contribution to the theoretical literature on bargaining and voting in bankruptcy. So far theoretical studies mostly focused on bargaining within complete information framework (see, for example, Baird and Picker (1991), Bebchuck and Chang (1992) and Eraslan (2006)). An exception is Kordana and Posner (1999) but they do not look at the information aggregation issues. The papers within bankruptcy literature that are most closely related this work are, Bond and Eraslan (2006), which analyzes the optimal voting arrangements among claimants when the proposals are given endogenously; and Maug and Yılmaz (2002), which shows that two class voting may be optimal when there are conflicts of interest among claimants (but takes the proposals to be exogenous).

The rest of the paper is organized as follows. In the next section, we describe our

\footnote{When one agent owns the 100% of the good, the problem reduces to the bilateral bargaining case of Myerson and Satterthwaite (1983).}
model. In Section 3 we present our analysis: in Section 3.1 we analyze the case where operational restructuring and financial restructuring is decided simultaneously, and in Section 3.2 we analyze the sequential cases. Section 4 concludes. Appendix provides the omitted proofs.

2 The Model

We consider a firm that is in Chapter 11 bankruptcy. There are two claimants of the firm. The creditor, denoted by $c$, owns a debt contract against the firm with face value $F > 0$, and the equityholder, denoted by $e$, is the residual claimant. For expositional clarity, the creditor is male and the equityholder is female. Both of the claimants are risk-neutral and maximize the expected value of their payoffs. The firm value is endogenous and depends on the “business plan” of the firm chosen by the claimants. Consequently, the claimants need to decide both which business plan to implement (operational restructuring) and how to split the value (financial restructuring).

There are a finite number of possible business plans. The set of business plans is denoted by $\{0, 1, \ldots, J\}$. In addition to depending on the business plan deterministically, the value of the firm also depends on the state of the world randomly. Let $R^j$ denote the random firm value when business plan $j$ is chosen. There are two equally likely states of the world, $G$ and $B$ and $R^j$ takes the value $R^j_\sigma$ when the state $\sigma$ is realized. We refer to plan 0 as the liquidation plan and denote the liquidation value of firm in state $\sigma$ by $L_\sigma$, and so $L_\sigma = R^0_\sigma$.

If an agreement cannot be reached and the firm is liquidated then proceeds are distributed according to the absolute priority rule. That is, the creditor is paid first up to the amount he is owed and the rest of the proceeds are paid to the equityholder. Since the liquidation value is random, the liquidation payoffs are also random. We let $\tilde{L} = \tilde{R}_0$ denote the random liquidation value and $\tilde{\ell}$ denote the random payoff to the
creditor in the event of liquidation. If the original claims are in place when liquidation
takes place, then we have $\tilde{\ell} = \min\{\tilde{L}, F\}$. Thus, the expected payoff of the creditor
and the equityholder in the event of disagreement are given by $E[\tilde{\ell}]$ and $E[\tilde{L} - \tilde{\ell}]$
respectively.

We assume that both the reorganization and liquidation values are higher in the
good state, i.e. $R^j_G > R^j_B$ for all $j$. If there exists plans $j$ and $j'$ such that $R^j_G \leq R'^j_G$ and
$R^j_B \leq R'^j_B$, it is never efficient to implement plan $j$ since plan $j'$ results in a higher firm
value regardless of the state of the world.\footnote{Specifically, we will focus on the most efficient equilibrium and such restructuring plans will never be part of such equilibria.} We rule out such trivial cases and assume
without loss of generality that the plans are enumerated in the order of increasing
volatility. Therefore, we have $R^j_G > R^{j-1}_G > \ldots > R^1_G > R^0_G = L_G > L_B = R^0_B >
R^1_B > \ldots > R^{J-1}_B > R^J_B$.

At the beginning of the game, each claimant receives a private signal about the state
of the world. Private signals are independently drawn from a common state-dependent
distribution. The support of claimant $i$’s signal is denoted by $S_i = [0, 1]$. Let $\phi_i(s_i|\sigma)$
denote the density function for signal $s_i$ conditional on the true state of the world being
$\sigma$. We assume that $\phi_i(s_i|\sigma)$ is continuous and satisfies monotone likelihood ratio property
(MLRP) so that $\frac{\phi_i(s_i|G)}{\phi_i(s_i|B)}$ is strictly increasing in $s_i$. In other words, higher signals
are more likely conditional on the good state. A direct implication of these assump-
tions is that $\Pr(G|s_c, s_e)$ is continuous and strictly increasing in $s_c$ and $s_e$. In addition,
we assume that $\phi_i(0|G) = 0$ and $\phi_i(1|G) = \infty$, i.e. extreme signals reveal the state of the
world perfectly. This implies that with sufficiently low signals liquidation maximizes
the expected value of the firm conditional on the signals, and with sufficiently high
signals, another restructuring plan is optimal. Thus, our assumption that the extreme
signals reveal the state of the world perfectly makes the problem non-trivial in the
sense that the aggregation of private information matters in terms of maximizing the
expected value of the firm.\footnote{Note that MLRP alone does not imply that information aggregation is necessary for efficiency.}

An immediate implication of MLRP is that conditional on a pair of signals if plan $j'$ yields a higher expected value than a less risky plan $j$, then when the claimants receive more optimistic signals, it is still the case that the expected firm value under plan $j'$ is higher than that of under plan $j$. Consequently, as either $s_c$ or $s_e$ increases, more volatile business plans become more and more attractive in terms having a higher expected firm value.

**Lemma 1** If $E[\tilde{R}^j'|s_c, s_e] > E[\tilde{R}^j|s_c, s_e]$ for some $j', j$ with $j' > j$ and signal pair $s_c, s_e$ then $E[\tilde{R}^j'|s'_c, s'_e] > E[\tilde{R}^j|s'_c, s'_e]$ for all $s'_c \geq s_c$ and $s'_e \geq s_e$.

An important component of our analysis is that financial restructuring allows issuance of new securities in return for the cancelation of the old claims. In other words, unlike the standard bargaining models where proposals are restricted to the division of a cake using cash payments, here the proposals may specify the division of many cakes (one for each possible realization of the firm value) using payments in securities. Specifically, we assume that the set of feasible proposals is given by $\mathcal{P} = \{h: R \rightarrow R|0 \leq h(R) \leq R, \forall R\}$ with functions in $\mathcal{P}$ specifying the feasible securities that can be offered to the creditor. Thus the creditor’s payoff is a function of the realized firm value and the equityholder receives the difference between the firm value and the payoff to the creditor. The restrictions we impose on the set of feasible proposals, i.e. $0 \leq h(R)$ and $h(R) \leq R$, follow from the limited liability constraints for the creditor and the equityholder respectively. In all other aspects, they are general enough and includes various standard securities that are used in practice such as cash in the amount $C$: $h(R) = C$, debt with face value $D$: $h(R) = \min\{D, R\}$; and fraction $\alpha$ of the equity: $h(R) = \alpha R$ among other standard securities.

As mentioned earlier, we are interested in comparing the outcomes when operational
Restructuring and financial restructuring are decided simultaneously, when operational restructuring is decided first, and when financial restructuring is decided first. Consequently, in our analysis we need to look at three different extensive forms. First consider the simultaneous game in which the operational and financial restructuring are decided at the same time. The extensive form of this game is as follows. After the signals are realized the creditor makes an announcement \( \mu : S_c \rightarrow \mathcal{M}_c \). Without loss of generality, we take \( \mathcal{M}_c \) to be equal to \( S_c \). Having seen the creditor’s message, the equityholder proposes a business plan \( j : \mathcal{M}_c \times S_e \rightarrow \{0, \ldots, J\} \) and a financial restructuring \( \pi : \mathcal{M}_c \times S_e \rightarrow \mathcal{P} \). Next both claimants simultaneously vote on the proposal bundle. If both claimants vote to accept, then both the business plan and the financial restructuring are implemented. Otherwise the firm is liquidated and the proceeds are distributed according to the absolute priority rule under the original claims, i.e. the creditor is paid the minimum of the proceeds and his claim \( F \), and the equityholder received the rest, if any.

Next, consider the sequential game in which operational restructuring is decided first, and the financial restructuring is decided second. The extensive form of this game is as follows. First, the creditor sends his message \( \mu : S_c \rightarrow \mathcal{M}_c \). Then, the equityholder proposes a business plan \( j : \mathcal{M}_c \times S_e \rightarrow \{0, \ldots, J\} \). Both claimants simultaneously vote on the business plan. If it is accepted by both claimants, then it is implemented after the financial restructuring stage. Otherwise, the firm is liquidated after the financial restructuring stage.\(^9\) Once the votes \( v_i : \mathcal{M}_c \times S_i \times \{0, \ldots, J\} \rightarrow \{a, r\} \) are observed, the equityholder proposes a financial restructuring \( \pi : \mathcal{M}_c \times S_e \times \{0, \ldots, J\} \times \{a, r\} \times \{a, r\} \rightarrow \mathcal{P} \). A second vote is taken, this time on the financial restructuring. If both claimants vote to accept the financial restructuring, then it is implemented. Otherwise, the old claims stay in place.

\(^9\)This is inessential, it can be liquidated immediately.
Finally, we consider the sequential game in which financial restructuring is decided first, and the operational restructuring is decided second. The extensive form of this game is as follows. First, the creditor sends his message $\mu : S_c \rightarrow M_c$. Then, the equityholder proposes a financial restructuring $\pi : M_c \times S_e \rightarrow \mathcal{P}$. Both claimants simultaneously vote on the financial restructuring. If it is accepted by both claimants, then it is implemented. Otherwise, the old claims stay in place. Once the votes $v_i : M_c \times S_i \times \mathcal{P} \rightarrow \{a, r\}$ are observed, the equityholder proposes a business plan $j : M_c \times S_e \times \mathcal{P} \times \{a, r\} \times \{a, r\} \rightarrow \{0, \ldots, J\}$. A second vote is taken, this time on the business plan. If both claimants vote for the business plan, then it is implemented. Otherwise, the firm is liquidated.

We restrict attention to pure strategy profiles which consists of (i) a message strategy for the creditor, (ii) one or two proposal strategies for the equityholder depending on whether the issues are negotiated simultaneously or sequentially, and (iii) voting strategies for both claimants following each proposal. The equilibrium concept we use is Perfect Bayesian Equilibrium which consists of a strategy profile and a set of beliefs for both claimants which are obtained by Bayesian updating whenever possible, and the strategies are sequentially rational. As it is common for games of incomplete information there will be multiple equilibria and we will focus on efficient equilibrium. Consequently, our benchmark will be the first best. We say that an equilibrium is efficient (first best) if expected firm value is maximized for all possible signal pairs. Let $j^*(s_c, s_e)$ denote the optimal business plan when the signal pair is $(s_c, s_e)$, that is, $j^*(s_c, s_e)$ satisfies $E[\tilde{R}^j(s_c, s_e)|s_c, s_e] \geq E[\tilde{R}^j|s_c, s_e]$ for all $j$, i.e.,

$$\Pr(G|s_c, s_e)R^j_G(s_c, s_e) + \Pr(B|s_c, s_e)R^j_B(s_c, s_e) \geq \Pr(G|s_c, s_e)R^j_G + \Pr(B|s_c, s_e)R^j_B,$$

for all $j \in \{0, 1, \ldots, J\}$. Thus, an equilibrium is efficient if and only if $j^*(s_c, s_e)$ is the business plan agreed upon in equilibrium for all $s_c, s_e$. 

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3 Analysis

We start by establishing results that are applicable to all the extensive forms discussed.
Given an equilibrium of any extensive form described above, let $j(s_c, s_e)$ and $\hat{\pi}(s_c, s_e)$
denote the business plan and financial restructuring agreed upon in equilibrium when
the signal pair is $s_c, s_e$. First, we establish that in order to have an efficient equilib-
rium, the creditor must be able to communicate his information and he should do so
truthfully:

**Lemma 2** In every efficient equilibrium the creditor reports his signal truthfully for
all $s_c > 0$: If $j(m_c, s_e) = j^*(s_c, s_e)$ for all $(s_c, s_e)$ then $\mu(s_c) = s_c$ for all $s_c > 0$.

To see the intuition, notice that for any two distinct signals $s_c, s'_c$ of the creditor,
there exists a signal $s_e$ such that when the equityholder’s signal is $s_e$, the optimal
restructuring plan is different under $s_c$ and $s'_c$, i.e., $j^*(s_c, s_e) \neq j^*(s'_c, s_e)$. Therefore,
for the equityholder to be able to propose the optimal restructuring plan, the creditor
has to communicate his information to the equityholder. Since the creditor has only
finite number of actions available to him without communication, but his signals can
take a continuum of values, it is immediate that efficiency requires communication.

**Proposition 1** In any game without communication, an efficient equilibrium outcome
does not exist.

What is not immediate is whether or not communication itself is sufficient for existence
of an efficient equilibrium. Recall that in a standard mechanism design approach, the
mechanism designer commits to an outcome for any given vector of messages. Even
then, efficiency is not always attained for the problem we study.\(^{10}\) As such it is natural
that communication is not sufficient for existence of an efficient equilibrium when one

\(^{10}\)See for example, Fieseler, Kittsteiner and Moldovanu (2000).
restricts attention to a particular bargaining game. As we will see below, security design coupled with communication may, however, be sufficient for efficiency depending on the extensive form of the bargaining game.

As we have seen, the creditor must report his signal truthfully in an efficient equilibrium. Therefore, his equilibrium payoff function \( \hat{\pi}(s_c, s_e) \) must satisfy the following condition:

**Lemma 3** Every efficient equilibrium satisfies

\[
E[\hat{\pi}(s_c, s_e)(\tilde{R}^*_{s_c, s_e})]|s_c| \geq E[\hat{\pi}(s'_c, s_e)(\tilde{R}^*_{s'_c, s_e})]|s_c| \quad \text{for all } s_c.
\]

We next analyze the equityholder’s decision. Efficient operational restructuring clearly requires the proposal of the optimal business plan \( j^* > 0 \). Furthermore, the equityholder must have an incentive not to propose \( j > 0 \) when it is optimal to liquidate. The following lemma identifies a set of necessary conditions for these two to hold.

**Lemma 4** If \( j^*(s_c, s_e) \neq j^*(s'_c, s'_e) = 0 \) for some \( s_c, s_e \) and \( s'_c, s'_e \), and the equilibrium is efficient, then it must be the case that

\[
E[\tilde{R}^*_{s_c, s_e} - \hat{\pi}(s_c, s_e)(\tilde{R}^*_{s_c, s_e})]|s_c, s_e| \geq E[\tilde{L} - \hat{\ell}|s_c, s_e|]
\]

and

\[
E[\tilde{L} - \hat{\ell}|s_c, s'_e| \geq E[\tilde{R}^*_{s_c, s_e} - \hat{\pi}(s_c, s_e)(\tilde{R}^*_{s_c, s_e})]|s_c, s'_e|.
\]

**Proof:** If an equilibrium is efficient, then for any signal pair \((s_c, s_e)\) such that \( j^*(s_c, s_e) > 0 \) the optimal restructuring plan must be proposed and voted for in that equilibrium. This requires that the equityholder must propose the optimal restructuring plan and vote for the plan regardless of the extensive form whenever the optimal restructuring plan is not liquidation. Therefore, her expected payoff in doing so, \( E[\tilde{R}^*_{s_c, s_e} - \hat{\pi}(s_c, s_e)(\tilde{R}^*_{s_c, s_e})]|s_c, s_e| \) must exceed the expected payoff under liquidation, \( E[\tilde{L} - \hat{\ell}|s_c, s_e| \). Thus, equation (3) must hold in an efficient equilibrium. Similarly,
in an efficient equilibrium the equityholder must not have an incentive to implement 
\( j > 0 \) when liquidation is optimal. This implies equation (4).

Note that although these two conditions are necessary, they are not sufficient for 
the equityholder to propose the optimal business plan. In particular, the conditions as 
to why the equityholder prefers \( j^* > 0 \) to another \( j > 0 \) is not stated above. When 
we characterize an efficient equilibrium, these additional conditions will need to be 
satisfied as well.

Next, we establish the necessity of financial restructuring in obtaining the most 
efficient outcome in equilibrium. In the absence of financial restructuring, the simult-
aneous and the sequential games are equivalent. Therefore, the following proposition 
holds for both.

**Proposition 2** Without financial restructuring, an efficient equilibrium exists if and 
only if \( F \geq R_G \) or \( F \leq R_B \).

**Proof:** Note that if there is no financial restructuring then any financial restruc-
turing proposal must leave the claims unchanged. Then it must be the case that 
\( \hat{\pi}(s_c, s_e)(R) = \min\{R, F\} \) for all \( s_c, s_e \).

We first show that if an efficient equilibrium exists without financial restructuring 
then \( F \geq R_G \) or \( F \leq R_B \). Suppose not. Then \( \hat{\pi}(s_c, s_e)(R) = \min\{R, F\} \) and \( R_B < \ \\
F < R_G \). Since \( R_B < F < R_G \), it follows that either \( L_B \leq F < R_G \) or \( L_G \geq F > R_B \) 
(or both).

First consider the case in which \( L_B \leq F < R_G \). Recall that for sufficiently high 
signals \( s_c \) and \( s_e \), the optimal restructuring plan is \( J \), i.e. \( j^*(s_c, s_e) = J \). Consequently, 
under optimal restructuring plan, we have \( \hat{\pi}(s_c, s_e)(R_G) = F \) and \( \hat{\pi}(s_c, s_e)(R_B) = R_B \). 
Furthermore, \( L_\sigma - \ell_\sigma = \max\{0, L_\sigma - F\} \). Therefore, equation (4) in Lemma 4 reduces to

\[
\Pr(G|s_c, s'_e) \max\{0, L_G - F\} \geq \Pr(G|s_c, s'_e)(R_G - F).
\]

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This cannot hold since $R_G^J > L_G$ and $R_G^J > F$.

Next, consider the case in which $L_G \geq F > R_B^J$. Note that for any $s_e$, there exists $s_c$ sufficiently high and $s_c'$ sufficiently low such that $j^*(s_c, s_e) \neq 0$ and $j^*(s_c', s_e) = 0$. Since the equilibrium is efficient and there is no financial restructuring, it must be the case that $\hat{\pi}(s_c, s_e)(R_G^J(s_c, s_e)) = F = \hat{\pi}(s_c, s_e)(R_G^J(s_c', s_e)) = \hat{\pi}(s_c', s_e)(L_G)$ and $\hat{\pi}(s_c, s_e)(R_B^J(s_c, s_e)) < \min\{L_B, F\} = \hat{\pi}(s_c, s_e)(R_B^J(s_c', s_e)) = \hat{\pi}(s_c', s_e)(L_B)$. Therefore, the creditor always prefers liquidation. Consequently, equation (2) in Lemma 3 cannot be satisfied.

To complete the proof, we need to show that if either $F \geq R_G^J$ or $F \leq R_B^J$ then an efficient equilibrium exists. In the former case, the creditor is always paid in full and has no incentive to misrepresent his signal or vote against the optimal restructuring proposal. Since the equityholder receives the full surplus she proposes the optimal restructuring plan. In the latter case, the equityholder would receive nothing in liquidation. Therefore, she is indifferent between the optimal restructuring plan and any other plan. Thus proposing the optimal restructuring plan and then voting for it is an equilibrium.

The proof of Proposition 2 illustrates that absent financial restructuring there are conflicts of interests between the claimants when the face value of debt is at an intermediate value. In particular, if $L_G > F$ then the creditor is always better off by liquidating the firm, and if $F > L_B$ then the equityholder is always worse off by liquidating the firm. As such the equilibrium is least likely to be efficient when the $L_G > F > L_B$. In what follows, we will assume this is the case.

### 3.1 Simultaneous Game

In the simultaneous game, if liquidation is proposed, then it takes place regardless of how the claimants vote. However, if the creditor is offered a liquidation with payoff
less than what he would have received under the existing financial structure, he will not approve the proposal and force liquidation under the original financial structure. Similarly, the equityholder will not propose liquidation together with a financial restructuring that decreases her payoff. Therefore, regardless of the offer, in equilibrium we have $\ell_G = F$ and $\ell_B = L_B$. Consequently, in order to minimize notation, without loss of generality, we assume that: (i) whenever liquidation is proposed, it allocates the claimants exactly their original liquidation payoffs; (ii) whenever liquidation is proposed, the claimants vote for it, (iii) whenever the equityholder is indifferent between proposing something else followed by voting against it and proposing liquidation, she does the latter. The equilibrium payoffs and outcomes do not depend on whether these assumptions are satisfied or not because in either case the claimants receive their original liquidation payoffs. These assumptions effectively allow us to substitute the notation for the equilibrium proposals in place of the disagreement payoffs.

So far we have used $\pi$ to denote the creditor’s payoff function in the event of financial restructuring. However, in the simultaneous game financial restructuring is specified along with a specific business plan (operational restructuring). Consequently, $R$ and thus, $\pi(\cdot)(R)$ can take only two values. In this case, two securities, debt and equity, can fully specify the financial restructuring. Let $f_i(s_c, s_e)$ and $\alpha_i(s_c, s_e)$ denote the claimant $i$’s face value of debt and equity share when signal pair is $(s_c, s_e)$.\footnote{Note that since the creditor must report his signal truthfully in equilibrium we replaced his report with his signal for clarity.}

Our next result has two equally important components. First, in a simultaneous negotiation game an efficient equilibrium always exists. Second, in any efficient equilibrium all of the new debt is allocated to the creditor.

**Proposition 3** In the simultaneous game with communication and security design first an efficient equilibrium exists. In every efficient equilibrium $f_c(s_c, s_e) \in [R_B^{\ast}(s_c, s_e), R_G^{\ast}(s_c, s_e)]$.
$L_G + F$ and $f_e(s_c, s_e) = 0$ for all $(s_c, s_e)$, i.e., the financial restructuring allocates all the debt to the creditor.

From Proposition 2, we know that if $F \in (R_G^l, R_B^l)$, then equilibrium is always inefficient absent financial restructuring. Therefore, existence of an efficient equilibrium in the presence of financial restructuring highlights the role of security design in conflict resolution. In particular, only the financial restructurings that preserve the relative seniority facilitates efficiency. To see the intuition, suppose that contrary to our result, an efficient restructuring can assign a fraction of the senior security to the equityholder when a (non-liquidation) restructuring is optimal. More specifically, consider a non-liquidation proposal that offers the equityholder a positive fraction of the senior security following creditor’s truthful message of $s_c$. Now consider the decision problem of the equityholder with $s_e = 0$ following $s_c$. Any efficient equilibrium requires the liquidation to take place. However, the equityholder’s expected payoff is zero under liquidation. This is because whenever liquidation takes place under simultaneous negotiations, any financial restructuring must preserve the original claims. Therefore, the equityholder gets a non-zero payoff only in good state which has probability zero given $s_e = 0$. Consequently, he would be better off deviating and proposing a business plan together with a financial restructuring in which he receives some of the senior security. This contradicts the efficiency since liquidation does not take place even though it is optimal.

Our result on efficient financial restructurings has direct, testable implications on financial structure in firms exiting Chapter 11 proceedings. In particular, our analysis suggest that efficient operational restructurings are accompanied by financial restructurings in which the relative priority is preserved. That is no senior securities must be allocated to senior claimants first, and junior claimants should receive their payments mostly in terms of junior securities. Although there is no study that has directly documented this fact, results presented in Franks and Torous (1989) is highly suggestive of
preserving initial seniority in successful reorganizations. For example, of the payment
to the banks, 49.70% is in the form of senior debt. In contrast, senior debt constitutes
0.86% of the payments to the trade creditors. Likewise, although preferred stockholders receive 29.94% of their recoveries in the form of warrants, secured debtholders never receive warrants.

Note that efficient financial restructuring is not unique in the sense that the creditor’s new debt has a face value that ranges from $R^*_{j(s_c, s_e)}$ to $R^*_{G(s_c, s_e)} - L + F$. Our next result is a direct implication of our previous result showing what happens if we restrict total face value in a reorganized firm to the lowest possible level.

**Corollary 1** In equilibrium, a financial restructuring that minimizes $f_c(s_c, s_e) + f_e(s_c, s_e)$ for all $(s_c, s_e)$ achieves efficiency if and only if $f_c(s_c, s_e) = R^*_{j(s_c, s_e)}$ and $f_e(s_c, s_e) = 0$ for all $(s_c, s_e)$, i.e., the new debt is lower for more volatile reorganizations.

The above corollary implies that firms that emerge from Chapter 11 proceedings with riskier business plans should have lower fraction of debt. Of course, many other models would generate such a prediction. Recall, however, that what we refer to as “debt” is the most senior security in the capital structure. More generally, our model predicts that firms with riskier business plans should have lower fractions of senior securities. Consequently, for an all equity firm that has common stock and warrants in its capital structure, we would expect to see larger fraction of warrants if the post-bankruptcy operations of the firm is riskier.

Another direct implication of Proposition 3 is on the division of the surplus. As we have seen, $\hat{\pi}(s_c, s_e)(R^*_B) = R^*_{j(s_c, s_e)}$ so that the creditor receives the entire reorganization value in the bad state. This implies that in a reorganized firm the equityholder receives a positive payoff only in the good state. On the other hand, she receives $L - F$ in good state and 0 in bad state under liquidation. She must have at least the same expected payoff for her to propose and accept a reorganization plan. Therefore, her
payoff in good state must be at least \( L_G - F \). However, if she receives more in good state under reorganization, then she will have an incentive to prevent liquidation even when liquidation is optimal. Consequently:

**Corollary 2** In any equilibrium of the simultaneous game, the creditor receives all the surplus.

### 3.2 Sequential Game

In a sequential game, the financial restructuring and operational restructuring are decided sequentially. We first analyze the case in which the financial restructuring takes place after operational restructuring.

In this case, our results from the simultaneous case changes remarkably. In particular, efficiency can no longer be achieved in equilibrium.

**Proposition 4** In the sequential game in which operational restructuring is decided first, there does not exist an efficient equilibrium.

**Proof:** Take any \( s_c, s_e \) such that \( j^*(s_c, s_e) > 0 \), and suppose that the optimal plan \( j^*(s_c, s_e) \) is proposed and accepted prior to any financial restructuring. Then the equityholder will neither offer nor accept a financial restructuring that makes him worse off than the existing financial structure:

\[
\Pr(G|s_c, s_e)[R^*_{G}(s_c, s_e) - \hat{\pi}(s_c, s_e)(R^*_{G}(s_c, s_e))] + \Pr(B|s_c, s_e)[R^*_{B}(s_c, s_e) - \hat{\pi}(s_c, s_e)(R^*_{B}(s_c, s_e))] \\
\geq \Pr(G|s_c, s_e)[R^*_{G}(s_c, s_e) - F].
\]

Therefore, the creditor can get at most

\[
\Pr(G|s_c, s_e)F + \Pr(B|s_c, s_e)R^*_{B}(s_c, s_e).
\]
But this is strictly less than

$$\Pr(G|s_c, s_e)F + \Pr(B|s_c, s_e)L_B,$$

what he can get in liquidation. Therefore, the creditor never votes for a reorganization plan. ■

Once the operational restructuring is decided, there is no financial restructuring that results in any wealth transfers in expected terms: any financial restructuring that is more desirable than the status quo (i.e. the original claims) to one of the claimants is less desirable than the status quo for the other party. Anticipating this incentive problem, the creditor never agrees to an operational restructuring in the first place.

Next we consider the game in which the financial restructuring takes place prior to the business restructuring.

Define

$$p_j^G(s_c) = \Pr(j^*(s_c, s_e) = j, \sigma|s_c), \quad p_j^B(s_c) = \Pr(j^*(m_c, s_e) = j, \sigma|s_c)$$

and

$$q(s_c) = \Pr(\sigma = G|s_c) = \sum_{j=0}^J p_j^G.$$ Conditional on creditor’s signal $s_c$, define the expected surplus by

$$\Delta(s_c) = \sum_{j=0}^J p_j^G(s_c)(R_j^G - L_G) - p_j^B(s_c)(L_B - R_j^B).$$

This expression represents the expected firm value under the optimal operational restructuring over and above the liquidation value conditional on the creditor’s information. Note that the definition of optimal business plan $j^*$ implies that

$$p_j^G(s_c)(R_j^G - L_G) \geq p_j^B(s_c)(L_B - R_j^B)$$

for all $s_c$. Since $j = 0$ denotes the liquidation alternative, this holds with equality if and only if $j = 0$. It follows that $\Delta(s_c) = 0$ if $s_c = 0$, and $\Delta(s_c) > 0$ for all $s_c > 0$.

The assumptions that $\phi_i(s_i|\sigma)$ is continuous and satisfies MLRP imply that $\Delta(s_c)$ is a continuous function and is strictly increasing. We can also decompose the expected surplus into benefit and cost,

$$\sum_{j=0}^J p_j^G(s_c)(R_j^G - L_G)$$

and

$$\sum_{j=0}^J p_j^B(s_c)(L_B - R_j^B),$$

respectively.
Finally, we define the expected surplus conditional on creditor’s signal $s_c$ when the creditor’s message is $m_c$

$$\Delta(s_c|m_c) = \sum_{j=0}^J p^j_G(s_c|m_c)(R^j_G - L_G) - p^j_B(s_c|m_c)(L_B - R^j_B),$$

where $p^j_x(s_c|m_c) = \Pr(j^*(m_c, s_e) = j, \sigma|s_c)$.

In our next proposition, we show that not only an efficient equilibrium exists but also the equityholder’s surplus can be bounded away from zero. There are several such equilibria, each associated with a different financial restructuring. Our proof is constructive and we characterize one such equilibrium that involves a simple financial restructuring. From the construction of the equilibrium, it will be clear that the equityholder’s surplus can be higher at the expense of a more complicated financial restructuring. As we show in Lemma 5 in the appendix, there exists $K \in (0, F - L_B)$ that satisfy

$$\sum_{j=1}^J p^j_G(s_c)K < \Delta(s_c) \text{ for all } s_c > 0.$$

In the following proposition the left hand-side of this inequality will be the equityholder’s expected surplus.

**Proposition 5** In the sequential game in which financial restructuring is decided first, efficiency can be achieved by a financial restructuring in which equityholder’s expected surplus is strictly positive.

**Proof:** We construct an efficient equilibrium. The equilibrium we construct is as follows: In period (i), the creditor’s message is truthful, $m_c = \mu(s_c) = s_c$. In period (ii), if $m_c = 0$, then the equityholder proposes a financial restructuring that preserves the original claims, i.e., $\pi(m_c, s_e)(R) = \min\{F, R\}$ for all $m_c, s_e$ with $m_c = 0$. Otherwise,
she proposes $\pi : \mathcal{M}_c \times S_e \rightarrow \mathcal{P}$ where for all $m_c$ and $s_e$

$$
\pi(m_c, s_e)(R) = \begin{cases} 
R & \text{if } R < F - K \\
F - K & \text{if } F - K \leq R \leq L_G \\
F - K + R - L_G & \text{if } R > L_G.
\end{cases}
$$

(5)

In period (iii) both claimants approve the new financial restructuring. In period (iv) the equityholder proposes $j^*(m_c, s_e)$ and in period (v) both claimants approve. For simplicity we fix the off the beliefs equilibrium path such that creditor (equityholder) believes that $s_e = 0$ ($s_c = 0$).

We now verify that these beliefs support the above equilibrium outcome. We first consider the case in which $s_e \neq 0$. If no deviations are detected until the last period, then the equityholder is indifferent between liquidation and $j^*$ as his expected payoff is $Pr(G|s_c, s_e)(L_G - F + K)$ under the new claims. Since the equityholder is indifferent, the creditor captures the entire surplus by voting for $j^*$. Consequently, the creditor’s expected payoff goes down by $\Delta(s_c)$ if he votes against so he approves as well. Note that in period (iv), the creditor cannot detect a deviation as long as any operational restructuring other than liquidation is proposed. If on the other hand liquidation is proposed, the vote is immaterial as liquidation will take place whether or not it is voted for.

As in period (v), the equityholder’s expected payoff is constant in period (iv) over all possible deviations and equilibrium strategies she may have as long as there has been no prior deviation. Thus, she has no incentive to deviate as long as no deviation has occurred. If there was a deviation in period (iii), at least one of the claimants must have voted against the financial restructuring and hence the claims are not restructured. This is possible only when at least one of the claimants believes that liquidation is optimal. There are three possibilities: either $s_c = 0 < s_e$, or $s_c = 0 < s_e$, or $s_c = s_e = 0$. If $s_c = 0$, then regardless of what the equityholder believes, there will be liquidation,
since the creditor’s expected payoff is maximized under liquidation with the original financial structure. If on the other hand, $s_c > 0$ and $s_e = 0$, the equityholder may propose a restructuring other than liquidation. But since claims are not restructured, the creditor’s expected payoff when he votes against financial restructuring in period (iii) is $qF + (1 - q)L_B$. In contrast, his expected payoff if he votes for the financial restructuring in period (iii) is

$$q[F - K + \frac{\Delta(s_c)}{q}] + (1 - q)L_B$$

if he had not deviated. Since $L_B \leq F - K$ and $K < \frac{\Delta(s_c)}{q}$, the creditor does not deviate in period (iii) as long as there has been no prior deviation. The equityholder approves in period (iii) as well, given that, if he deviates, he gets $Pr(G|s_c, s_e)(L_G - F)$ in comparison to $Pr(G|s_c, s_e)(L_G - F + K)$.

In period (ii) the equityholder does not deviate for the same reason: if she deviates, her expected payoff is $Pr(G|s_c, s_e)(L_G - F + K)$ in comparison to $Pr(G|s_c, s_e)(L_G - F)$. Note that at this stage she cannot detect a deviation.

Finally, in period (i) the creditor’s message is truthful. To see this first note that reporting $m_c = 0$ results in original financial structure being preserved followed by liquidation. Thus, when $m_c = 0$, the expected payoff of the creditor is $qF + (1 - q)L_B$ which is strictly less than $q[F - K + \frac{\Delta(s_c)}{q}] + (1 - q)L_B$ as we argued above. Reporting another untruthful message $m_c \neq s_c$ will result in an expected payoff of $q[F - K + \frac{\Delta(s_c|m_c)}{q}] + (1 - q)L_B$. Note that $\Delta(s_c|m_c)$ is maximized at $m_c = \mu(s_c) = s_c$. Therefore, this deviation is not profitable either.

For the case with $s_c = 0$, the expected surplus $\Delta(s_c) = 0$ and thus deviations are not profitable for the creditor. Consequently, the equityholder proposes liquidation and the firm is liquidated under original claims.

This result highlights the importance of the order in which issues are negotiated. Although an efficient equilibrium exists when financial restructuring is decided first,
there is no efficient equilibrium when operational restructuring is decided first. This is because, once operational restructuring is decided, there is no incentive to transfer wealth between the claimants. Anticipating this lack of financial restructuring, the claimants do not have any incentive to share information and agree on the optimal reorganization at the first stage. On the other hand, changing the order and deciding on financial restructuring first enables the claimants to credibly communicate their information. In particular, it is possible to design a financial restructuring that would reward the claimants when the optimal restructuring plan is implemented at a later date, and punish them otherwise.

The following corollary shows that the financial restructuring that enables efficiency in the simultaneous game, can also lead to an efficient operational restructuring in a sequential game.

**Corollary 3** *In the sequential game in which financial restructuring is decided first, efficiency can also be achieved by a financial restructuring that allocates all of the senior security and the expected surplus to the creditor.*

In addition to its implications for efficiency, the order of negotiations also has distributional consequences. In particular, any financial restructuring that achieves efficiency under simultaneous negotiations also achieves efficiency under sequential negotiations (in which financial restructuring is decided first), but the converse is not true. More specifically, it is possible that the equityholders receive a positive surplus. Furthermore, it is also possible that the equityholders receive part of the senior securities. Note that under sequential negotiations, it is possible that an agreement is reached on financial restructuring first, but no agreement is reached at the operational restructuring stage. This is due to the fact that the equityholder does not commit to a specific organizational restructuring at the time of the financial restructuring. As such, liquidation can take place under claims that differ from the original claims. Therefore,
there is one less condition for efficient equilibrium payoffs. This leads to a larger set of financial restructurings that achieve efficiency.

4 Conclusion

In this paper, we have analyzed a bargaining game between the senior and junior claimants of a firm in Chapter 11 bankruptcy. The claimants are asymmetrically informed about the desirability of various operational restructuring alternatives, and negotiate to decide both what to do with the firm (operational restructuring) and how to split the surplus (financial restructuring). We show that without communication or security design, it is not possible to have an efficient equilibrium. When both communication and security design is allowed, whether efficiency can be achieved or not depends on the order in which two issues (operational restructuring and financial restructuring) are negotiated. When the issues are negotiated simultaneously, there exists an efficient equilibrium. The financial restructuring that supports an efficient outcome requires the senior claimants to receive senior securities. When the issues are negotiated sequentially, there does not exist an efficient equilibrium if operational restructuring is negotiated first. On the other hand, if financial restructuring is negotiated first, an efficient equilibrium exists. Our results highlight the sensitivity of the ex post efficiency of Chapter 11 bankruptcy to the order in which the operational restructuring and financial restructuring decisions are made.

In our analysis, we restricted attention to the case in which offers specify what to do with the firm and who gets what. It is also possible that the offer in the first stage specifies the identity of the proposer in the second stage. Our results would be unchanged even when we modify the strategy space to allow this.

To see note first consider the case in which operational restructuring is decided first. Now suppose that the offer in the first stage specifies what to do with the firm as well
as who proposes the financial restructuring. As before, once operational restructuring is agreed upon, there is no financial restructuring that is acceptable to both claimants. Hence, regardless of the identity of the proposer, the original claims are preserved at the financial restructuring stage, and being able to propose at the second stage has no value. Given this, the creditor does not agree to any operational restructuring other than liquidation at the first stage.

Second, consider the case in which financial restructuring is decided first. For this case, we have seen that when the equityholder proposes in both stages, an efficient equilibrium exists. It can easily be seen that, in the modified game, there exists an equilibrium in which the first stage proposal specifies the equityholder to be the proposer for the second stage.

Finally, note that our analysis focused on a single take-it-or-leave-it offer. It is possible to extend our analysis by allowing negotiations to continue several periods before liquidation takes place, and letting players discount the future payoffs. The main implication of such an extension would be on the sharing of the surplus. In particular, the equityholders would receive some of the surplus, even in the simultaneous game.

Appendix

Proof of Lemma 1: We have $R^*_j > R^j_G > R^j_B > R^*_B$ by definition. Let $\theta \equiv \Pr(G|s_e, s_c)$ and $\varphi \equiv \Pr(G|s'_e, s'_c)$ so that we have $\varphi \geq \theta$. Now $E[R|j^*, s_e, s_c] > E[R|j, s_e, s_c]$ is equivalent to $\theta R^*_G + (1 - \theta)R^*_B > \theta R^j_G + (1 - \theta)R^j_B$. Arranging terms results in $\theta[R^*_G - R^j_G] > (1 - \theta)[R^*_B - R^j_B]$. Given that $\varphi \geq \theta$ we must have $\varphi[R^*_G - R^j_G] > (1 - \varphi)[R^*_B - R^j_B]$. Therefore, $\varphi R^*_G + (1 - \varphi)R^j_B > \varphi R^*_B + (1 - \varphi)R^j_B$ must also hold. 

Proof of Proposition 3: Recall from Lemma 4 that if $j^*(s_c, s_e) \neq j^*(s_e, s'_c) = 0$ for
some \( s_c, s_e \) and \( s'_c \), and an efficient equilibrium exists, then it must be the case that

\[
E_\sigma[R_j^{\ast} - \hat{\pi}(s_c, s_e)(R_{\sigma}^{\ast})|j^\ast (s_c, s_e), s_c, s_e] \geq E_\sigma[(L_\sigma - \ell_\sigma) |s_c, s_e]
\]

and

\[
E_\sigma[(L_\sigma - \ell_\sigma) |s_c, s'_e] \geq E_\sigma[R_j^{\ast} - \hat{\pi}(s_c, s_e)(R_{\sigma}^{\ast})|j^\ast (s_c, s_e), s_c, s'_e].
\]

Using \( \ell_G = F \) and \( \ell_B = L_B \) yields

\[
\Pr(G|s_c, s_e)[R_G^{j^\ast(s_c, s_e)} - \hat{\pi}(s_c, s_e)(R_G^{j^\ast})] + \Pr(B|s_c, s_e)[R_B^{j^\ast(s_c, s_e)} - \hat{\pi}(s_c, s_e)(R_B^{j^\ast})] \geq \Pr(G|s_c, s_e) (L_G - F)
\]

and

\[
\Pr(G|s_c, s'_e)(L_G - F) \geq \Pr(G|s_c, s'_e)[R_G^{j^\ast(s_c, s_e)} - \hat{\pi}(s_c, s_e)(R_G^{j^\ast})] + \Pr(B|s_c, s'_e)[R_B^{j^\ast(s_c, s_e)} - \hat{\pi}(s_c, s_e)(R_B^{j^\ast})].
\]

We divide the first inequality by \( \Pr(G|s_c, s_e) \) and the second by \( \Pr(G|s_c, s'_e) \):

\[
[R_G^{j^\ast(s_c, s_e)} - \hat{\pi}(s_c, s_e)(R_G^{j^\ast})] + \frac{\Pr(B|s_c, s_e)}{\Pr(G|s_c, s_e)}[R_B^{j^\ast(s_c, s_e)} - \hat{\pi}(s_c, s_e)(R_B^{j^\ast})] \geq (L_G - F), \quad (6)
\]

and

\[
(L_G - F) \geq [R_G^{j^\ast(s_c, s_e)} - \hat{\pi}(s_c, s_e)(R_G^{j^\ast})] + \frac{\Pr(B|s_c, s'_e)}{\Pr(G|s_c, s'_e)}[R_B^{j^\ast(s_c, s_e)} - \hat{\pi}(s_c, s_e)(R_B^{j^\ast})]. \quad (7)
\]

Note that \( j^\ast(s_c, s_e) > j^\ast(s_c, s'_e) \) implies that \( s_e > s'_e \), and hence \( \frac{\Pr(B|s_c, s'_e)}{\Pr(G|s_c, s'_e)} > \frac{\Pr(B|s_c, s_e)}{\Pr(G|s_c, s_e)} \). Therefore, we must have \( R_B^{j^\ast(s_c, s_e)} = \hat{\pi}(s_c, s_e)(R_B^{j^\ast}) \). Together with the inequalities (6) and (7) this implies that \( f_j^{j^\ast(s_c, s_e)} \in [R_B^{j^\ast(s_c, s_e)}, R_G^{j^\ast(s_c, s_e)} - L_G + F] \) and \( f_j^{j^\ast(s_c, s_e)} = 0 \) when the new securities are restricted to only debt and equity. Considering the fact that for all \( j^\ast(s_c, s_e) \neq 0 \), there exists \( s'_e \) such that \( j^\ast(s_c, s'_e) = 0 \), we must have \( f_j^{j^\ast} \geq R_B^j \) and \( f_j^j = 0 \) for all \( j \in \{1, 2, \ldots, J\} \) as a necessary condition for an efficient outcome.
Next we construct an efficient perfect Bayesian equilibrium, and show that conditions $f_c^j(s_e,s_c) \in [R_B^j(s_e,s_c), R_B^g(s_e,s_c) - L_G + F]$ and $f_c^j(s_e,s_c) = 0$ are not only necessary but also sufficient. Let $\alpha^*_c(s_e,s_c) (R_B^g(s_e,s_c) - f_c^j(s_e,s_c)) = L_G - F$. In equilibrium, the creditor reveals his signal, $s_c$ correctly, and the equity holder proposes $j^*(s_e,s_c)$, $f_c^j(s_e,s_c) \in [R_B^j(s_e,s_c), R_B^g(s_e,s_c) - L_G + F], f_c^j(s_e,s_c) = 0$ and $\alpha^*_c(s_e,s_c) = \frac{L_G - F}{R_B^j(s_e,s_c) - f_c^j(s_e,s_c)}$ and $\alpha^*_c(s_e,s_c) = 1 - \alpha^*_c(s_e,s_c)$. If the equity holder proposes a business plan $j$ that is not accompanied by $f_c^j \in [R_B^j, R_B^g - L_G + F], f_c^j = 0$ and $\alpha^*_c = \frac{L_G - F}{R_G^j - f_c^j}$ and $\alpha^*_c = 1 - \alpha^*_c$, the creditor believes that $s_e = 0$. Note that the inequalities (6) and (7) imply that the expected payoff to the equityholder under restructuring is equal to that under liquidation. Therefore, given the equilibrium beliefs, the equityholder is indifferent and does not deviate. The creditor on the other hand captures the entire profit in equilibrium and thus cannot profitably deviate. ■

**Lemma 5** There exists $K \in (0, F - L_B)$ such that

$$\sum_{j=1}^{J} \mathcal{P}_c^j(s_c) K < \Delta(s_c) \text{ for all } s_c > 0.$$  

**Proof:** For a given $s_c$, let $s_e(s_c)$ satisfy

$$(R_B^1 - L_G) \Pr(\sigma = G|s_e(s_c)) = (L_B - R_B^1) \Pr(\sigma = B|s_e(s_c)).$$

In other words, for a given $s_c$, $s_e(s_c)$ is the signal that makes the expected firm value equal under liquidation and business plan $j = 1$. Note that $\frac{\Pr(\sigma = G|s_e(s_c))}{\Pr(\sigma = B|s_e(s_c))} = \frac{\phi_e(s_e(s_c)|G)}{\phi_e(s_e(s_c)|B)} \frac{\phi_e(s_e(s_c)|G)}{\phi_e(s_e(s_c)|B)}$. Therefore, $s_e(s_c)$ must satisfy

$$\frac{\phi_e(s_e(s_c)|G)}{\phi_e(s_e(s_c)|B)} \frac{\phi_e(s_e(s_c)|G)}{\phi_e(s_e(s_c)|B)} = \frac{L_B - R_B^1}{R_B^G - L_G} \text{ for all } s_c > 0.$$ 

Given that $\frac{\phi_e(s_e(s_c)|G)}{\phi_e(s_e(s_c)|B)}$ is strictly increasing we must have

$$\frac{\phi_e(s_e(s_c)|G) \int_{s_e(s_c)}^{1} \phi_e(s|G) ds}{\phi_e(s_e(s_c)|B) \int_{s_e(s_c)}^{1} \phi_e(s|B) ds} > \frac{L_B - R_B^1}{R_B^G - L_G} \text{ for all } s_c > 0.$$
Note that left hand side of the above inequality is \( \sum_{j=1}^{J} \frac{p_{j}^{G}(s_c)}{\sum_{j=1}^{J} p_{j}^{B}(s_c)} \). Arranging the terms, we have
\[
R_{ij} - L_G - (L_B - R_{Bi}) \frac{\sum_{j=1}^{J} p_{j}^{B}(s_c)}{\sum_{j=1}^{J} p_{j}^{G}(s_c)} > 0 \text{ for all } s_c > 0.
\]
Furthermore, from the definition of \( \Delta(s_c) \), the left hand side of the above inequality is strictly less than \( \frac{\Delta(s_c)}{\sum_{j=1}^{J} p_{j}^{G}(s_c)} \) for all \( s_c > 0 \). Therefore, \( \frac{\Delta(s_c)}{\sum_{j=1}^{J} p_{j}^{G}(s_c)} \) is strictly positive for all \( s_c > 0 \). Thus, \( K \in (0, \frac{\Delta(s_c)}{\sum_{j=1}^{J} p_{j}^{G}(s_c)}) \) exists for all \( s_c > 0 \).

**Proof of Corollary 3:** The proof follows immediately from that of Proposition 5 once \( K \) is replaced with zero.

**References**


