The Life Insurance Demand in a Heterogeneous-Agent Life Cycle Economy

Ning Wang
Department of Risk Management and Insurance
Georgia State University
October 6th, 2010

Abstract

A household’s life insurance demand depends on the household characteristics and the economic situation. In this paper, we construct a life cycle model of married households with market wages and mortality shocks to quantitatively analyze the life insurance demand by heterogeneous households. In this model, parents are altruistic towards each other as well as their children, and choose consumption, working hours, and life insurance holdings to maximize their expected lifetime utility. The peak of the household’s life insurance demand in the model economy is on average $370,000 occurring at age 33. Our results suggest that the most important determinants of life insurance demand are financial vulnerability, the amount of financial support needed and life insurance premium. We find that the peak of life insurance demand for single-parent households is well before couple households. Moreover, increasing the number of children attributes a large increase of life insurance demand in single-parent households, but has no significant effect on couple households. We also discuss the impact of wage shock on joint decision of life insurance purchases between couples: one’s good wage shock results in an increase of one’s working hours and life insurance demand, but a decrease of spouse’s working hours and life insurance demand.

Keywords: life insurance demand; heterogeneous households; life cycle model
I Introduction

By holding term life insurance, a household can hedge against the decline in total household income resulting from the death of a wage-earner, and parents can provide financial security for dependents after their death. In this paper, we construct a heterogeneous-agent life cycle model to explore the relation of a household’s life insurance demand to its specific household characteristics and the economic situation. Household characteristics here include marital status, the amount of wealth, the number of children, household income, household age, risk attitude and so on.

There is no consensus about the amount and the distribution of household life insurance holdings in empirical research since the data of household life insurance is limited. Chambers, Schlagenhauf, and Young (2003) examine life insurance data from SCF\(^1\) for 1995, 1998 and 2001, and find that the peak of life insurance holdings is on average $250,000\(^2\) occurring around 50 years old. Lin and Grace (2007) examine SCF data for 1992, 1995, 1998, and 2001, and show the mean of life insurance holdings in selected data is $366,263\(^3\). They find that the most significant relationship between life insurance holdings and financial vulnerability is among younger households from age 20 to 34. In addition, they find that older households from age 50 to 64 tend to use less life insurance to protect a certain level of financial

---

\(^1\) Survey of Consumer Finances  
\(^2\) $250,000 in 2001 can be adjusted to be $302,000 in 2009 by inflation rates.  
\(^3\) Their study shows the median, the 75\(^{th}\) upper tail, the mean, and the maximum of term life insurance holdings in their selected data are $56,700, $227,250, $366,263 and $80,000,000 respectively.
vulnerability than middle-aged households from age 35 to 49. LIMRA\(^4\) (2007) investigates that the average life insurance coverage needed is $459,000 while the average life insurance owned is $126,000. In this paper, we construct a model to quantitatively analyze life insurance holdings in a household, and compare our results with these empirical studies.

It is a fact that SCF data contains information only on the total amount of life insurance held by each household, and not on the division of life insurance between couples. In this paper, we develop a heterogeneous-agent life cycle model to be able to analyze the joint life insurance purchases decision of married couples.

Hong and Rios-Rull (2007) build an OLG model to analyze social security, life insurance and annuities for households. In their model, they assume agents have a bequest motive, and focus on the implications of social security under different baseline economies rather than life insurance demand. Chambers, Schlagenhauf, and Young (2009) construct an OLG model to find an economic puzzle that life insurance holdings simulated in their model are much larger than their observed data in Chambers, Schlagenhauf, and Young (2003). The peak of life insurance holdings is twice as much as their empirical study in 2003, occurring at age 30 instead\(^5\). Nishiyama (2010) develop an OLG model with uninsurable wage shocks to analyze the effect of spousal and survivors benefits on the labor supply of married couples. In this paper, we extend this OLG model to focus on life insurance demand of married couples.

\(^4\) Life Insurance Marketing and Research Association

\(^5\) They choose a small value of risk aversion (1.5) in their model to provide a lower bound for estimation.
We construct a life cycle model of married households with market wages and mortality shocks to quantitatively analyze the life insurance demand for heterogeneous households. In the model, we introduce the number of children by ages calibrated by USA data into household characteristics. Parents are altruistic towards each other as well as their children, and choose consumption, working hours, and life insurance holdings in each year to maximize their expected lifetime utility. As a result, the peak of household life insurance holdings in our model is around $370,000 occurring at age 33. We also discuss reasons why the life insurance demand in our model turns out to be higher than previous empirical studies.

Then we explore the impact of some factors in benchmark economy in the model on life insurance demand. Firstly, the life insurance demand by household ages suggests that the most important determinants of life insurance demand are financial vulnerability, the amount of financial support needed and life insurance premium. We find that financial vulnerability is the primary determinant of life insurance demand in a household during its early ages when the household has low wage earning and saving wealth; while life insurance premium is the primary determinant during its late ages when the household faces highest mortality risk. Secondly, our results show that the peak of life insurance demand for single-parent households is well before couple households. In addition, increasing birth rate attributes a large increase of life insurance demand in single-parent households, but has no significant influence on couple households. Finally, we discuss joint decision of life insurance purchases between couples in couple households: if one receives good wage shock, she/he will
increase her/his labor time and life insurance coverage, but her/his spouse will relatively decrease his/her labor time and life insurance coverage.

The paper is structured as follows. Section II develops heterogeneous household life cycle model in detail. Section III is the model calibration by USA data. Section IV shows main results of the model, and discusses how some benchmark factors affect the life insurance demand. Section V provides current conclusions and future research works.

II The Model

In this part, we build a life cycle model to quantitatively derive the optimal decisions of life insurance holdings, consumption expenditure and labor time in a household, and focus on the optimal decision-making for life insurance purchases among heterogeneous households.

1 Heterogeneous Households

The households are heterogeneous with respect to several factors in this economy. One factor is household age, denoted by \( k = k_{\text{min}}, k_{\text{min}} + 1, \ldots, k_{\text{max}} \). The household can enter the economy in the model when husband’s age is over 20. For simplicity, we assume that the husband and the wife in a household are at the same age, and never get divorced. The number of children in each household is related to household age in the calibration. Note that the child in the model refers to a kid who is still younger
than 20 years old. The wage rates per efficient unit of labor for each gender, $w_1$ and $w_2$, also vary at different household ages. Another two heterogeneous factors are husband’s and wife’s working ability, denoted by $e_1$ and $e_2$ respectively, both of which are assumed to follow the markov process and are independent from each other. The beginning-of-period household wealth, $a$, is another heterogeneous factor. It changes in accord with household optimal saving decisions in last period. The parameter $m$ has four values to specify four heterogeneous martial statuses among households: the married-couple household if $m=0$, the single-father household if $m=1$, the single-mother household if $m=2$, and the kids-only household if $m=3$. In this model, we assume that all households are married couples at the very beginning, and calibrate marital status movement among heterogeneous households over life cycle time based on the data of mortality rate.

Let $s$ be the individual state vector of a household in the model, $s = (a, e_1, e_2, m, k)$.

### 2 Household’s Utility Function

The household’s utility function in each year is a Cobb-Douglas and CRRA function, and depends on its current marital status $m$ and the number of children $n$.

Firstly, utility functions of single-parent households ($m=1$ or $m=2$) are as follows,

$$U(c, l_1; n, m=1) = \left( \frac{c}{(1+n/2)^{\eta}} \right)^{\alpha} \frac{(l_1^{1-\gamma})^{1-\gamma}}{1-\gamma}$$

$$U(c, l_2; n, m=2) = \left( \frac{c}{(1+n/2)^{\eta}} \right)^{\alpha} \frac{(l_2^{1-\gamma})^{1-\gamma}}{1-\gamma}$$
where $c$ is household consumption at household age $k$; $l_1$ is leisure time of husband, which is set to be 1 in the single-mother family; $l_2$ is leisure time of wife, which is set to be 1 in the single-father family; $n$ is the number of children calibrated in this household year; $\eta$ is the index of the economy scale between 0 and 1 and it implies that two-adult household spends $2^\eta$ times as much as a one-adult household for the same level of living standard; the child-adult equivalency factor is $1/2$; the relative risk aversion is $\gamma$; $\alpha$ and $(1-\alpha)$ are elasticity of consumption and leisure time in utility function.

Then the couple household’s utility function ($m=0$) is the summation of two utility functions above with slight modification,

$$U(c, l_1, l_2; n, m=0) = \left[ \frac{\frac{c}{(2+n/2)^{\eta}}}{(l_1)^{1-\alpha}} \right]^{1-\gamma} + \left[ \frac{\frac{c}{(2+n/2)^{\eta}}}{(l_2)^{1-\alpha}} \right]^{1-\gamma}$$

where the number of equivalent adults in a couple household becomes $(2+n/2)$. Finally, the utility function of children-only households ($m=3$) is,

$$U(c; n, m=3) = n * \left[ \frac{c}{\frac{n}{2}^{\eta}} \right]^{1-\gamma}$$

where the number of equivalent adults in a kids-only household becomes $n/2$; leisure time for mother and father are both set to be 1; and all consumption commodities are going into children’s part. Here we set the economy scale $\eta$ to be 1 since we assume...
that children do not share consumption commodities without parents’ custody. Then we sum up all equivalent adults’ utility to get total utility for the children-only household.

3 Household’s Optimization Problem

In our model, household fund sources for each year are wages, life insurance payment if death occurs and social security payment after retirement, and the fund is annually distributed into three categories: consumption expenditure, saving wealth, and life insurance purchases. The household chooses the optimal decision path for each specific fund source and fund usage to maximize its expected utility over life cycle time.

We assume that initial households are all married couples with some children at age 20. In year $k$, each household receives working ability $e_1$ for husband and $e_2$ for wife, and faces mortality rates $(1-\varphi_{1,k})$ for husband and $(1-\varphi_{2,k})$ for wife. To maximize expected lifetime utility of a household, the adults will choose the following optimal decision rules together in each year: household consumption $c$, husband’s leisure time $l_1$, wife’s leisure time $l_2$, end-of-period saving wealth $\bar{a}$, husband’s life insurance coverage $d_1$ and wife’s life insurance coverage $d_2$. In this model, $d_1$ and $d_2$ are viewed as life insurance demand of the husband’s and the wife’s. Here we assume that life insurance can be purchased at the actuarially-fair price.

If a household becomes a single-father household with a certain probability, the amount of wife’s life insurance payment $d_2$ will be added into household saving.
wealth. The single-father will then choose the optimal decision rules each year to maximize household utility in the following life cycle time: household consumption \( c \), end-of-period saving wealth \( \tilde{a} \), his leisure time \( l_2 \), and his life insurance coverage \( d_1 \). The single-mother household in the model follows the same logic.

If a household unfortunately becomes a kids-only household, the children will only choose the amount of household consumption and end-of-period saving wealth to maximize household’s utility.

Let \( V(s) \) be value function of the household in individual state \( s \). The optimization problem for the household in life cycle model should be as follows,

\[
V_k(s) = \max \left[ U(c, l_1, l_2; n_k, m) + \beta \int V_{k+1}(s') \, d\Pi_k(e_1', e_2', m'|e_1, e_2, m) \right]
\]

(1) Control variables’ constraints,
\[
c > 0; \\
0 < l_1 \leq 1; 0 < l_2 \leq 1; \\
d_1 \geq 0; d_2 \geq 0; \\
l_1=1, d_1=0, \text{if } m=2 \text{ or } 3; \\
l_2=1, d_2=0, \text{if } m=1 \text{ or } 3;
\]

(2) The law of motion of household end-of-period wealth,
\[
\tilde{a} = (1+r) a + w_{1,k} e_1 (1-l_1) + w_{2,k} e_2 (1-l_2) + \mathbb{1}_{k \geq 65} \mathbb{1}_{m \leq 3} (1+\mathbb{1}_{m=0}) ss - c - (1-\varphi_{1,k}) d_1 - (1-\varphi_{2,k}) d_2;
\]

(3) The law of motion of household beginning-of-period wealth,
\[
a' = \tilde{a}(s) + d t, \text{if } m=1 \text{ and } m'=3 \text{ or } m=0 \text{ and } m'=2;
\]
\( a' = a(s) + d_2 \text{ if } m=2 \text{ and } m' = 3 \text{ or } m=0 \text{ and } m' =1 \);\\
\( a' = a(s) + d_1 + d_2 \text{ if } m=0 \text{ and } m' =3 \);\\
\( a' = a(s) \), otherwise;\\

(4) The law of motion of household state variables, \( s' = (a', e_1', e_2', m', k+1) \).

where \( r \) is the interest rate; \( w_{1,k} \) is husband’s wage rate per efficient unit of labor at age \( k \); \( w_{2,k} \) is wife’s wage rate per efficient unit of labor at age \( k \); \( l_m=0, l_{k>65} \) and \( l_{m<3} \) are all indicator functions; \( ss \) is social security payment per person above 65 years old; \( \phi_{1,k} \) is survival rate for husband at the end of age \( k \); \( \phi_{2,k} \) is survival rate for wife at the end of age \( k \); \( \Pi_k (e_1', e_2', m' | e_1, e_2, m) \) is the state transition probability function in the optimization, which would be calibrated in section III.

4 Household distribution function

For population aggregation, we construct population distribution function for households in different states. The household states can include saving wealth amount, husband/wife’s working ability, marital status and household age.

Let \( x(s) \) be the household population probability density function at age \( k \), and let \( X(s) \) be the corresponding cumulative distribution function.

The household population for each age is normalized to unity,

\[
\sum_{m=0}^{3} \int_{A \times E^2} dX(s) = 1, \text{ where } s = (a, e_1, e_2, m, k)
\]
The law of motion of household population distribution is as follows,

\[ x(s') = \sum_{m=0}^{3} \int_{A^2} 1 \{ a' = a' (a(s), m') \} \Pi_k (e_1, e_2, m|e_1, e_2, m) \ dX(s); \]

where \( s' = (a', e_1', e_2', m', k+1) \).

Then aggregated saving wealth, consumption and life insurance demand for each household age are as follows,

\[
W_k = \sum_{m=0}^{3} \int_{A^2} a \ dX(s)
\]

\[
C_k = \sum_{m=0}^{3} \int_{A^2} c(s) \ dX(s)
\]

\[
D_k = \sum_{m=0}^{2} \int_{A^2} [d_1(s) + d_2(s)] \ dX(s)
\]

### III Calibration

In this section, we calibrate the model to match pertinent USA data. Since whether wealth distribution is consistent with USA data is essential in the model, our calibration focuses on preference parameters, demographic distribution and income distribution.

1 **Economic parameters**

Table 1 is a list of main economic parameters in this model. These parameters’ values are all consistent with either economic literature or USA historical data.
2 The number of children

In this model, we assume that all households have the same number of children for each age. We use the average number of children estimated by Nishiyama (2010, unpublished Excel table). Figure 1 shows the fertility rate by mothers’ age, and Figure 2 shows the estimated average number of children used in this paper.

3 State Transition Probability Function

(1) state transition function

For simplicity, we assume that both husband’s working ability and wife’s working ability are independent of his/her mortality rate. The state transition function for households can be obtained by the following formula,

\[
\Pi_k (e_1', e_2', m' | e_1, e_2, m) = \Pi_k (m' | m) \Pi (e_1' | e_1) \Pi (e_2' | e_2)
\]

(2) Marital status transition process

In this life cycle model, all households are initially married couples. With certain probabilities and evolving paths, initial couple-households turn to be heterogeneous with different marital statuses over life cycle time. To simply specify probabilities of marital status movement, we assume that husband’s mortality rate and wife’s mortality rate are independent of each other. The household marital status transition probability matrix from state \(m\) at age \(k\) to state \(m'\) at age \((k+1)\) is \(\Pi_k (m' | m)\).
Survival rates by ages are cited from Table 4 of 2010 Annual Statistical Supplement in Social Security Administration. Figure 3 shows the population distribution with respect to marital status calibrated in the model.

(3) Working ability transition process

With idiosyncratic wage shocks for each household state, wage-earners have access to different levels of working ability in the model. The motions of husband’s working ability $e_1$ and wife’s working ability $e_2$ are both assumed to follow Markov chains. The stochastic processes of $e_1$ and $e_2$ are as follows,

$$\ln e_{1,j+1} = \ln e_{1,j} + z_{1,j+1}$$

$$\ln z_{1,j+1} = \rho z_{1,j} + \varepsilon_{1,j+1} \quad \text{where} \quad \varepsilon_{1,j} \sim \mathcal{N}(0, \sigma_1^2)$$

$$\ln e_{2,j+1} = \ln e_{2,j} + z_{2,j+1}$$

$$\ln z_{2,j+1} = \rho z_{2,j} + \varepsilon_{2,j+1} \quad \text{where} \quad \varepsilon_{2,j} \sim \mathcal{N}(0, \sigma_2^2)$$

where $z_1$ and $z_2$ are persistent wage shocks of men and women.

Nishiyama (2010) calibrates income distribution for men and women by estimating these parameters above, which is consistent with the data from 2009 weekly earnings in CPS$^6$. This approximation yields the vector of persistent wage shock nodes and the

$^6$ Current Population Survey
markov working-ability transition probability matrix for each gender as follows,

\[ z_1 = [0.3103, 0.5801, 1.0000, 1.7240, 3.2229] ; \]

\[ z_2 = [0.3322, 0.5988, 1.0000, 1.6701, 3.0099] ; \]

\[
\begin{pmatrix}
0.8979 & 0.1021 & 0.0000 & 0.0000 & 0.0000 \\
0.0308 & 0.8902 & 0.0790 & 0.0000 & 0.0000 \\
0.0000 & 0.0518 & 0.8964 & 0.0518 & 0.0000 \\
0.0000 & 0.0000 & 0.0790 & 0.8902 & 0.0308 \\
0.0000 & 0.0000 & 0.0000 & 0.1021 & 0.8979 \\
\end{pmatrix}
\]

\[ \Pi(e'_1|e_1) = \Pi(e'_2|e_2) = \]

4 Wage rates over life cycle time

Using the data of 2009 weekly earnings in CPS, we calibrate median wage rates for both husband and wife from age 21 to 65. Here people are assumed to be retired at age 65. The data shows that the median annual wage earning of a full-time employee for both genders is $739*52. Correspondingly in the model, median annual income is the product of benchmark wage rate per efficient unit of labor, median labor hours, and median working ability (1.0000). Here we assume that median labor time is 1/3 out of 1. So the benchmark wage rate in this economy should be \( \bar{w} = \frac{739*52}{(1/3)} / 1.0000 = 1.15284*10^5 \). We calculate the wage rate for each gender at each age by multiplying the benchmark wage rate \( \bar{w} \) and the ratio of his/her median weekly wage at each age to the median full-time employee’s weekly wage ($739). Figure 4 shows
original and OLS-adjusted wage rates for both husband and wife over life cycle time.

Note that we cannot deal with the upper tail of income distribution and wealth distribution since our transition probability function cannot produce a very high income. Except the upper tail, income distribution by ages and wealth distribution by ages calibrated in the model are both relatively consistent with USA data.

**IV Outcomes**

Figure 5 shows the peak of life insurance demand in our model is around $370,000 occurring at age 33, and the demand begins decreasing quickly after age 54 by more than 10% per year. The life insurance demand in our model is higher than the observed data in chamber, Schlagenhauf and Young (2003). Firstly, we assume that all households are initially married couples, which can increase overall life insurance holdings since a single household in their economy should have relatively less life insurance demand. Secondly, saving wealth in our model is smaller than their data, and households tend to purchase more life insurance to protect its financial security. Considering wealth distribution in real world is skew and our model cannot produce a very high income, we calibrate wealth distribution using median value of household net worth in 2007 from SCF instead of mean value. The fact that the median value of net worth is less than the mean value leads to a small amount of saving wealth calibrated in our model. Finally, risk aversion coefficient is consistently equal to a high level of 4.0 over life cycle time in our model, which may push life insurance
demand up. If we change the coefficient of risk aversion into the same value (1.5) as Chambers, Schlagenhauf, and Young (2009) use, the peak of life insurance holding will decrease to $320,000 occurs at household age 37. Figure 6 provides life insurance demand with a low risk aversion. Our results are much smaller than their estimation and fit their observed data in 2003 better; however, it trades off wealth distribution in our model which is consistent with USA data.

We then explore the impact of some factors in benchmark economy in the model on life insurance demand.

1 Household ages

Figure 5 also shows the life insurance demand by household ages. During early ages from 20 to 30, households hold a small amount of wealth and relatively low earnings, and have an increasing number of children. They tend to save money and purchase a certain level of life insurance holdings to provide financial security. On one hand, households continually increase life insurance purchases to satisfy their increasing financial support needed. On the other hand, wage earners of young households have lowest mortality risk, so households choose to purchase high but not the highest life insurance coverage despite of super low life insurance premium. From age 31 to 40, households have the largest number of children and need a large amount of financial support from both saving wealth and life insurance purchases. Therefore, households keep purchasing high life insurance coverage to hedge mortality risk and wage shocks, and continue to save a lot to increase economic strength. The peak of life insurance
holdings occur at age 33. Since the number of children begins reducing from age 37, life insurance holdings starts decreasing by less than 3% per year in the late of this period. From age 40 to 54, the number of children in a household continues decreasing and wealth is continuously accumulated. Households decrease life insurance purchases by from 3% to 10% per year. Figure 3 shows there is a much larger chance for people above 55 years old to die than those below 55, and mortality shock is quickly increased after age 55. However, from age 55 to 65, households have almost no child and possess the largest amount of wealth to provide financial security. Considering life insurance premium is much more expensive for them than before, they largely decrease life insurance purchases by above 10% per year as a result.

The age distribution of life insurance holdings in the model suggests that the avoidance of financial vulnerability due to mortality shocks and wage shocks, the amount of financial support needed and life insurance premium are the most important determinants of life insurance demand.

2 Mortality shock

Figure 7 shows that life insurance demand greatly changes when we descend survival rates by 5% in the model. In this case, the household is not able to earn as much money as the benchmark economy, and household consumption and welfare are both dropping a lot. In spite of low income, households during early ages save much more

---

7 Although it is unrealistic that the survival rate can be reduced by 5%, we just test it to check the impact of mortality shock on life insurance demand.
money than the benchmark economy, and purchase much more life insurance coverage to provide financial security to protect households from higher mortality risk. In contrast, households tend to have significantly less life insurance holdings than the benchmark economy during late ages due to much higher life insurance premium.

Figure 8 further provides changes in life insurance demand for each gender. We can notice that wife’s life insurance holdings during early ages extraordinarily jump so much to make sure that she can hedge greater financial risk due to her higher death probability. We can also notice of a huge decreasing of life insurance holdings for husband among old households due to his much higher premium. Based on the fact that survival rate for wife is higher than husband in each household year, we can strengthen our inference that the avoidance of financial vulnerability is the dominating factor of life insurance purchases during household early ages and instead life insurance premium is the dominating factor during household late ages.

3 Marital status

Figure 9 shows life insurance demand with respect to specific marital status. Couple households follow the same tendency of life insurance demand movement as the whole population, which we discussed before. But we have a different story for single-parent households. The single-parent household tends to save much more than couple households at early ages to avoid potentially high financial vulnerability due to the last adult’s death. However, the peak of life insurance purchases for single-parent households (around age 28) is well before couple households (at age 33) since the
amount of financial support needed for single-parent households is smaller than that of couple households.

4 The number of children

Figure 10 shows the impact of the number of children in a household on life insurance demand. Compared to a slight growing up in couple household’s life insurance purchases with the increasing number of children, there is a big jump of life insurance holdings for single-parent households. Our results show that raising the number of children in each household by 10% leads to 12.8% higher life insurance demand of single-father households and even 15.6% higher life insurance demand of single-mother households. However, the life insurance demand of couple households increases only by 1.6% in this case. It suggests that increasing the number of children attributes a high increase of life insurance demand in single-parent households, but has no significant influence on couple households.

5 Household income and wealth

Figure 11 shows the relationship between husband’s life insurance demand and household wealth and his working ability at age 39. When husband’s working ability becomes higher, his life insurance demand is increased which can provides stronger financial security and guarantee that remaining household members are able to keep the same level of living standard before and after his death.

Figure 12 shows husband’s life insurance demand is reduced when wife’s working
ability is higher, which results from his prediction of less financial risk after his death.

Figure 13 shows joint decision of life insurance purchases between couples. It implies that the more important wage-earner holds higher life insurance coverage. It shows that with wife’s working ability increasing by 5%, husband’s life insurance demand reduces on average by 4.7% and wife’s life insurance demand grows up by 9.5%. Therefore, if one wage-earner receives good wage shock, she/he will increase her/his working hours and life insurance coverage, and her/his spouse will decrease his/her working hours and life insurance coverage to maximize household utility.

6 Household welfare

Figure 14 shows couple household’s welfare at age 39. It implies that household welfare is better off with working ability increasing and wealth rising. In future, we will check changes of household welfare by dismissing life insurance holdings in the model. Also we can check welfare changes by eliminating social security payment.

V Current conclusions and future research works

In this paper, we construct a heterogeneous-agent life cycle model with market wages and mortality shocks to explore the relation of a household’s life insurance demand to its specific household characteristics and the economic situation. The life insurance demand by ages in the model suggests the most important determinants of life insurance demand are financial vulnerability, the amount of financial support needed and life insurance premium. We also discuss the impact of marital status, the number
of children, mortality shocks and wage shocks on life insurance demand among heterogeneous households.

In future research, we will do sensitivity analysis to explore the impact of some other factors in the model economy on life insurance holdings, such as risk aversion, discount rate, social security and interest rate. We will try to examine SCF data for 2004 and 2007 to get wealth holding data and life insurance holding data, and compare our results with the observed data.

We also intend to construct Dynamic General Equilibrium model to simulate the life insurance demand in future, which involves heterogeneous households, life insurance companies, firms, and government.
References


Appendix

1 Algorithm

The state space of heterogeneous households in this life cycle model is $i_{max} * j_{max} * j_{max} * m_{max} * k_{max}$ (20 * 5 * 5 * 4 * 80). We solve household’s optimization problem backward from age $k_{max}$ with the assumption that the value function in the period after the last period, $V_{k_{max}+1}(s')$, is equal to 0.

Based on first-order conditions and the envelope condition, we construct Kuhn-Tucker conditions to figure out household’s optimal decision rules in state $s$, such as $c(s), l_1(s), l_2(s), d_1(s)$ and $d_2(s)$. Meanwhile, we update new household value $V_{k_{max}}(s)$ and marginal value $V_{k_{max},a}(s)$ for s-state household at corresponding time $k_{max}$. Then we use updated results for different household states in period $k_{max}$ to solve utility optimization problem at age $(k_{max} - 1)$. The optimal decision rules for each specific household state at each year can be dynamically solved in the same way.

2 Solve complementarity problems of household utility optimization

If $m = 0$, $V(a, m = 0) = U(c, l_1, l_2; n_b, m = 0) + \beta \{ \phi_{1k} * \phi_{2k} * V(a'; m' = 0, m = 0) + \phi_{1k} * (1 - \phi_{2k}) * V(a' + d_1; m' = 1, m = 0) + \phi_{2k} * (1 - \phi_{1k}) * V(a' + d_2; m' = 2, m = 0) + (1 - \phi_{1k}) * (1 - \phi_{2k}) * V(a' + d_1 + d_2; m' = 3, m = 0) \}$
\[ s.t. \quad a' = (1+r) a + w_{1,k} e_1 (1-l_1) + w_{2,k} e_2 (1-l_2) + \mathbb{1}_{k>65} \mathbb{1}_{l_2<3} (1+ \mathbb{1}_{m=0}) s_s - c - (1- \phi_{1,k}) d_1 - (1- \phi_{2,k}) d_2. \]

\[
U_c(c, l_1, l_2; n_k, m=0) = \lambda \quad (c)
\]

\[
U_{l_1}(c, l_1, l_2; n_k, m=0) = w_{1,k} e_1 * \lambda \quad (l_1)
\]

\[
U_{l_2}(c, l_1, l_2; n_k, m=0) = w_{2,k} e_2 * \lambda \quad (l_2)
\]

\[
\beta [ \phi_{1,k} \ast V_a(a'; m'=0, m=0) + (1- \phi_{1,k}) \ast V_a(a'+d_1; m'=2, m=0)] = \lambda \quad (d_1)
\]

\[
\beta [ \phi_{2,k} \ast V_a(a'; m'=0, m=0) + (1- \phi_{2,k}) \ast V_a(a'+d_2; m'=1, m=0)] = \lambda \quad (d_2)
\]

\[
\beta [ \phi_{2,k} \ast V_a(a'+d_1; m'=2, m=0) + (1- \phi_{2,k}) \ast V_a(a'+d_1+d_2; m'=3, m=0)] = \lambda (a')
\]

where \( \lambda \) is the Lagrangian Parameter.

If \( m=1 \),

\[
V(a; m=1) = U(c, l_1; n_k, m=1) + \beta [ \phi_{1,k} \ast V_a(a'; m'=1, m=1) + (1- \phi_{1,k}) \ast V_a(a'+d_1; m'=3, m=1)]
\]

\[ s.t. \quad a' = (1+r) a + w_{1,k} e_1 h_1 + \mathbb{1}_{k>65} s_s - c - (1- \phi_{1,k}) d_1; \]

\[
U_c(c, l_1; n_k, m=1) = \lambda \quad (c)
\]

\[
U_{l_1}(c, l_1; n_k, m=1) = w_{1,k} e_1 * \lambda \quad (l_1)
\]

\[
\beta \ast V_a(a'+d_1 ; m'=3, m=1) = \lambda \quad (d_1)
\]

\[
\beta \ast V_a(a'; m'=1, m=1) = \lambda \quad (a')
\]
If $m=2$,

$$V(a; m=2) = U(c, l_2; n_k, m=2) + \beta [\varphi_{2k} V(a'; m'=2, m=2) + (1-\varphi_{2k}) V(a'+d_2; m'=3, m=2)]$$

s.t. $a' = (1+r) a + w_2 h_2 + \|_{k>65} ss - c - (1-\varphi_{2k}) d_2$

$$U_c(c, l_2; n_k, m=2) = \lambda$$

$$U_{12}(c, l_2; n_k, m=2) = w_2 e_2 \varphi \lambda$$

$$\beta V_a(a'+d_2; m'=3, m=2) = \lambda$$

$$\beta V_a(a'; m'=2, m=2) = \lambda$$

If $m=3$,

$$V(a; m=3) = U(c; n_k, m=3) + \beta V(a'; m'=3, m=3)$$

s.t. $a' = (1+r) a - c$

$$U_c(c; n_k, m=3) = \lambda$$

$$\beta V_a(a'; m'=3, m=3) = \lambda$$
## Table 1

<table>
<thead>
<tr>
<th>Parameters</th>
<th>Notations</th>
<th>Values</th>
</tr>
</thead>
<tbody>
<tr>
<td>Interest rate</td>
<td>$r$</td>
<td>0.05</td>
</tr>
<tr>
<td>Time discount factor</td>
<td>$\beta$</td>
<td>0.94;</td>
</tr>
<tr>
<td>Share of consumption in utility</td>
<td>$\alpha$</td>
<td>0.36;</td>
</tr>
<tr>
<td>Index of household scale economies</td>
<td>$\eta$</td>
<td>0.678</td>
</tr>
<tr>
<td>Coefficient of relative risk aversion</td>
<td>$\gamma$</td>
<td>4.00</td>
</tr>
<tr>
<td>the number of wealth nodes</td>
<td>$imax$</td>
<td>20</td>
</tr>
<tr>
<td>the number of wage shock nodes</td>
<td>$jmax$</td>
<td>5</td>
</tr>
<tr>
<td>the number of marital status types</td>
<td>$mmax$</td>
<td>4</td>
</tr>
<tr>
<td>Initial household age</td>
<td>$kmin$</td>
<td>1</td>
</tr>
<tr>
<td>Retirement age</td>
<td>$kr$</td>
<td>45</td>
</tr>
<tr>
<td>Maximum household ages</td>
<td>$kmax$</td>
<td>80</td>
</tr>
<tr>
<td>Social security payment</td>
<td>$ss$</td>
<td>$16,500$</td>
</tr>
</tbody>
</table>
Figure 5

Family life insurance demand by ages for the whole population

Figure 6

Family life insurance demand changes by decreasing gamma
Figure 7

Life insurance demand changes by decreasing survival rates

Figure 8

Life insurance demand changes with respect to genders in lower survival rates
Figure 11

Figure 12
Figure 13

**life insurance demand changes by increasing wife's abilities**

Figure 14

**Household welfare for couples at age 39**