Changes in Risk and the Demand for Saving

Louis Eeckhoudt, Catholic University of Mons (Belgium) and CORE

Harris Schlesinger, University of Alabama

September 4, 2006
Abstract

This paper examines how stochastic changes in risk affect the demand for saving. We consider two models of savings demand: one in which future labor income is risky and one in which the return on savings is risky. In each model, we examine the effects of an $N^{th}$-degree stochastic change in the risk. For each case, we establish necessary and sufficient conditions on preferences that will guarantee that the individual increases his or her level of savings. We show how in the case of only labor income risk, any changes in savings are purely due to precautionary savings motives. For the case where the return on savings is risky, we show how both a precautionary effect and a substitution effect need to be analyzed.

**Keywords**: Edginess, Precautionary Savings, Prudence, Risk Apportionment, Risk Aversion, Stochastic Dominance, Temperance

**JEL classification**: D81
1 Introduction

In his famous treatise on *The General Theory of Employment, Interest, and Money*, John Maynard Keynes (1936) named a precautionary motive as one of three basic grounds for individuals to save money, rather than spend it on current consumption. This precautionary motive for savings is "[t]o provide for contingencies requiring sudden expenditure and for unforeseen opportunities of advantageous purchases ..." (Keynes, Chapter 13). In other words, precautionary savings was money put aside to deal with future uncertainty. It was not until a series of papers by Leland (1968), Sandmo (1970) and Dreze and Modigliani (1972) applied the newly developed (at the time) techniques of expected-utility theory that we had more formal models of Keynes’ notion. Finally, Kimball (1990) formalized the concept of "prudence," which generalized the main common hypotheses of these papers and developed what has become the modern theory of precautionary savings.
Although there are likely as many sources for this future uncertainty as there are individual consumers, one manifestation that has been used quite often in the economics and finance literatures has been an assumption that future labor income is risky. For the sake of concreteness, we will take this as the key source of risk. The theory of precautionary savings demand focuses on how savings behavior changes in the presence of this labor-income risk. One question that has not been asked, to the best of our knowledge, is how changes in this labor-income risk will alter the level of precautionary savings, *ceteris paribus*. In this paper, we ask how a stochastic deterioration in the distribution of future labor income will affect savings demand.

Another interesting question arises if we assume that the interest paid on savings is itself random, rather than fixed. For example, even in a world with "risk-free" bonds, such bonds are typically only risk-free with respect to any default risk. There is still a risk that the market rates may change. Rothschild and Stiglitz (1971) examined the consequence of a mean-preserving increase in interest-rate risk on the level of savings. Unfortunately, their result predated Kimball (1990), so that they were not able to relate their findings to prudence. A few recent textbooks do just that, however, and they portray the Rothschild and Stiglitz results in terms of relative prudence.\(^1\) But what of other types of stochastic changes in the interest

---

\(^1\)See, for example, Danthine and Donaldson (2005) and Eeckhoudt et al. (2005).
rate? What will be the effect on savings? We show how analysis of this case requires one to balance a substitution effect (possibly reducing savings if the return on savings becomes riskier) against a precautionary effect. If the precautionary motive is strong enough, in a way that we make precise in this paper, then savings will increase.

Our goal here is not to synthesize the literature on savings. Rather we take the two scenarios mentioned above and ask whether we can predict the qualitative change in precautionary savings due to a change in the relevant risk: either risky future labor income or a risky interest rate. In both cases, higher order effects of risk preferences (risk aversion, prudence, temperance, etc.) play a key role. Unlike the standard portfolio problem, which has very untidy necessary and sufficient conditions for reducing the proportion of risky assets in an optimal investment portfolio, the conditions for changes in savings demand are quite simple and quite intuitive.

We proceed by first examining the role of changes in the distribution of risky future labor income. We consider a deterioration in the distribution by way of both

---

2 Kimball (1992) has a very readable introduction to the topic and Gollier (2001) provides a nice introduction to precautionary motives with a general-equilibrium setting.

3 Necessary and sufficient conditions for first-order and second-order stochastic dominance shifts in a future labor income to decrease investment in the risky asset can be applied from Eeckhoudt, et al. (1996). We are not aware of necessary and sufficient conditions for the portfolio problem in the case where the return on market changes, though several sufficient conditions can be obtained. See, for example, Hadar and Seo (1990). Unfortunately, even these sufficient conditions are not very appealing, as discussed by Gollier (2001).
$N^{th}$-order stochastic dominance and an increase in $N^{th}$-degree risk, as defined by Ekern (1980). We then do the same for a change in the random return on savings.

2 Labor Income Risk and Precautionary Savings

The precautionary savings motive was first linked to the notion of "prudence" by Leland (1968), although the technical concept of "prudence" itself was first introduced by Miles Kimball (1990). We proceed by first setting up a simple model of precautionary savings. We consider a consumer with a two-period planning horizon. As a base case consider a consumer with a certain income stream of $y_0$ at date $t = 0$ and $y_1$ at date $t = 1$. The consumer must decide how much to save at date $t = 0$. Any income that is not saved is consumed at that date. There is a fixed rate of interest $r$ for both saving and for borrowing. Savings are allowed to be negative (i.e. we allow for borrowing), so long as the amount borrowed can be repaid from the future income $y_1$.

The consumer is assumed to have preferences that are intertemporally separable with a preference for smoothing consumption over time. To this end, the consumer chooses a level of savings so as to maximize her lifetime utility of consumption:

$$\max_s U(s) \equiv u(y_0 - s) + \frac{1}{1+\delta} u(y_1 + s(1+r)),$$  \hspace{1cm} (1)
where the utility function \( u \) is assumed to be strictly increasing and strictly concave and where \( \delta \) represents the consumer’s personal rate of discount for delaying the utility of future consumption. We also assume throughout this paper that \( u \) is continuously differentiable.

The first-order condition for (1) is

\[
U'(s) = -u'(y_0 - s) + \frac{1 + r}{1 + \delta} u'(y_1 + s(1 + r)) = 0,
\]

which we assume to hold at some interior value \( s^* \), \(-y_1 < s^*(1 + r) < y_0(1 + r)\). It also follows trivially from (1) that \( U(s) \) is strictly concave in \( s \), whenever the utility function \( u \) is strictly concave in consumption. As a result, the second-order condition for a maximum holds and \( s^* \) is unique. For example, if \( r = \delta \), then as is well known, savings is used as a device to perfectly eliminate fluctuations in consumption over time.

Now assume that labor income at date \( t = 1 \) is risky, say \( \tilde{y}_1 \), where \( E\tilde{y}_1 = y_1 \). In this case, it is not possible to perfectly smooth consumption over time. Indeed, consumption at date \( t = 1 \) depends on the realized value of future labor income \( \tilde{y}_1 \). Thus, the objective of the consumer becomes

\[
\max_s U(s) \equiv u(y_0 - s) + \frac{1}{1 + \delta} E u(\tilde{y}_1 + s(1 + r)).
\]
It follows in a straightforward manner from (3) that the optimal level of savings will necessarily increase whenever $Eu'(\tilde{y}_1 + s^*(1 + r)) \geq u'(y_1 + s^*(1 + r))$, which can be guaranteed for any arbitrary values of $y_1$ and $r$, and any arbitrary mean-$y_1$ random variable if an only if $u'$ is a convex function of consumption, i.e. if and only if $u''' \geq 0$. This increased level of savings due to the labor income risk is precisely the precautionary part of total savings.

Some insight into this result was provided long ago in a paper by Menezes and Hansen (1971), who showed how $u''' > 0$ is equivalent to the utility premium of Friedman and Savage (1948) being decreasing in wealth. The utility premium simply measures the difference in utility between $u(w)$ and $Eu(w+\tilde{x})$ for any random wealth variable $\tilde{x}$. Of course, if $E\tilde{x} \leq 0$, then this utility premium simply measures the loss of utility from adding the risk $\tilde{x}$ to one's wealth. As such, we can refer to this utility premium as a measure of "pain" for adding the risk $\tilde{x}$. Hence, prudence implies that the "pain" of the risk $\tilde{x}$ is lower at higher wealth levels. In the case of labor-income risk and savings, the individual can better cope with the risk $\tilde{y}_1$, even if the risk is not hedgeable, by shifting a bit more wealth from the riskless wealth at date $t = 0$ to the risky wealth (and hence risky consumption level) at date $t = 1$.

---

4We will consider an "increase" in savings to be in the weak sense that savings does not fall. All of our results below extend to strict inequalities by well known methods. However, this leads to more complicated mathematical conditions, with no real gain in economic insight. Hence, we consider only increases in savings in this weak sense throughout the paper.
This precautionary saving helps to alleviate the "pain" from the labor-income risk.\footnote{In another paper, Eeckhoudt and Schlesinger (2006), we use the utility premium as a measure of pain to characterize higher-order preference effects in much the same way as this characterization of prudence.}

3 Stochastic Changes in Labor-Income Risk

Obviously the standard precautionary-savings model as specified above is one example of a change in labor-income risk: from no risk to a risky labor income with no change in the mean income level. To generalize the analysis to more general risk changes, we consider the optimal level of savings for risky labor income \( \tilde{y}_1 \) that maximizes (3). We denote this optimal level of savings as \( s_y \). Thus, \( s_y \) is the solution to the first-order condition

\[
U'(s) = -u'(y_0 - s) + \frac{1 + r}{1 + \delta} Eu' \left( \tilde{y}_1 + s(1 + r) \right) = 0.
\]

Consider a stochastic change in random wealth at date \( t = 1 \) from \( \tilde{y}_1 \) to \( \tilde{x}_1 \).

Following Ekern (1980), we will characterize \( \tilde{x}_1 \) as an increase in \( N^{th} \)-degree risk over \( \tilde{y}_1 \) if \( \tilde{y}_1 \) dominates \( \tilde{x}_1 \) via \( N^{th} \)-order stochastic dominance and the first \( N - 1 \) moments for the distributions of \( \tilde{y}_1 \) and \( \tilde{x}_1 \) coincide. Thus, for example, \( \tilde{x}_1 \) is an increase in second-degree risk over \( \tilde{y}_1 \) if \( \tilde{y}_1 \) dominates \( \tilde{x}_1 \) via second-order stochastic
dominance and where both distributions have equal means. In other words, \( \tilde{x}_1 \) is what Rothschild and Stiglitz (1970) define as a mean-preserving increase in risk over \( \tilde{y}_1 \). Similarly, Menezes et al. (1980) describe a third-degree increase in risk, which they call an "increase in downside risk," which is equivalent to third-order stochastic dominance restricted to cases for which the means and variances of the two distributions are the same.

As shown by Ekern (1980), who generalizes the concepts from Rothschild and Stiglitz (1970) and from Menezes et al. (1980), \( \tilde{x} \) is an \( N^{th} \)-degree increase in risk over \( \tilde{y} \) if the distribution of \( \tilde{x} \) can be obtained from the distribution of \( \tilde{y} \) by a sequence of probability transformations that preserve the first \( N-1 \) moments of the distribution. By the definition of \( N^{th} \)-order stochastic dominance, this also requires (as a necessary, but not sufficient condition) that the \( N^{th} \) moment of the distribution be higher for \( \tilde{x} \) if \( N \) is even, and lower for \( \tilde{x} \) if \( N \) is odd. Based on the above characterization of an \( N^{th} \)-degree increase in risk, it is a tautology to now say that \( \tilde{y}_1 \) \( N^{th} \)-order stochastically dominates \( \tilde{x}_1 \), if \( \tilde{x}_1 \) can be obtained from \( \tilde{y}_1 \) via any sequence of increases in \( n^{th} \)-degree risk, for all positive integers \( n \leq N \).

Consider the simplest case, where \( \tilde{y}_1 = y_1 \) and \( \tilde{x}_1 = x_1 \) are both constants, with \( y_1 > x_1 \). It follows trivially from (2) that savings will increase whenever \( u' \) is decreasing, i.e. \( u'' \leq 0 \). But this also turns out to be a necessary and sufficient
condition for any stochastic change in labor income for which \( \tilde{y}_1 \) dominates \( \tilde{x}_1 \) by first-order stochastic dominance (FSD).

This will follow trivially from (4) whenever \( E u'(\tilde{y}_1+s_y(1+r)) \leq E u'(\tilde{x}_1+s_y(1+r)) \) for any arbitrary value of \( r \). Under first-order stochastic dominance, this follows for any arbitrary \( \tilde{y}_1 \) and \( \tilde{x}_1 \) exhibiting FSD if and only if \( u' \) is a decreasing function. In other words, a risk-averse consumer will always save more if risky future labor income becomes more risky in the sense of FSD.

But what about higher order stochastic changes in future labor income? It follows in the same manner as above that for any arbitrary degree of stochastic dominance \( N \), where \( \tilde{y}_1 \) dominates \( \tilde{x}_1 \) via \( N^{th} \)-order stochastic dominance (NSD), we obtain the following result. The result hinges on the following well known equivalence result:

**NSD Equivalence**

The following two statements are equivalent

(i) \( \tilde{y}_1 \) dominates \( \tilde{x}_1 \) via NSD

(ii) \( Ef(\tilde{y}_1) \leq Ef(\tilde{x}_1) \) for any arbitrary function \( f \) such that \( \text{sgn}(d^n f(t)/dt^n) = (-1)^n \), for all \( n = 1, 2, ..., N \).

We are now able to state the following result. For notational convenience, we denote \( f^{(n)}(t) \equiv d^n f(t)/dt^n \).

---

\(^6\) See, for example, Ingersoll (1987).
Proposition 1  Given a two-period consumption and savings problem as specified in (3), with risky future labor income \( \tilde{y}_1 \) or \( \tilde{x}_1 \), the following two statements are equivalent:

(i) The optimal level of savings under \( \tilde{x}_1 \) is always as least as high as under \( \tilde{y}_1 \), for every utility function \( u \) such that \( \text{sgn}[u^{(n)}(t)] = (-1)^{n+1}, \text{for } n = 1, 2, ..., N + 1 \)

(ii) \( \tilde{y}_1 \) dominates \( \tilde{x}_1 \) via NSD.

**Proof.** We require \( u' > 0 \) and \( u'' < 0 \) for (3) to be well defined. The result here follows by simply defining \( f(t) \equiv u'(t+s_y(1+r)) \) in the NSD equivalence statements, where \( s_y \) is the solution to (4). ■

Note that in the "standard" model of precautionary savings, in which we go from a fixed future labor income to a random one, we can think of \( \tilde{y}_1 = y_1 \) and \( E\tilde{x}_1 = y_1 \). Thus, this is just a special case of Proposition 1. Since precautionary savings are zero when the future wealth is not risky, we obtain that \( u''' \geq 0 \) is both necessary and sufficient for the existence of precautionary savings.

Although not much has been written about higher-order effects, there are a few papers. The condition that \( u^{(4)}(t) \leq 0 \) is often referred to as "temperance" (Kimball, 1992) and the condition \( u^{(5)}(t) \geq 0 \) is referred to as "edginess" (Lajeri-Chaherli, 2004). Although these authors find such conditions as necessary for various types of behavior, they do not offer any settings under which these conditions are sufficient.
for anything. Eeckhoudt and Schlesinger (2006) provide a characterization of preferences, which is equivalent to \( \text{sgn}(u^{(n)}(t)) = (-1)^{n+1} \) for all \( n = 1, 2, ..., N \), which they define as satisfying "risk apportionment of order \( N \)." But they do not offer any implications of their generalization for optimizing behavior.

Proposition 1 can be easily extended to the following Corollary, which follows in a straightforward manner by using Ekern’s definition of an increase in \( N \)th-degree risk.

**Corollary 1:** Given a two-period consumption and savings problem as specified in (3), with risky future labor income \( \tilde{y}_1 \) or \( \tilde{x}_1 \), the following two statements are equivalent:

(i) The optimal level of savings under \( \tilde{x}_1 \) is at least as high as under \( \tilde{y}_1 \), for every utility function \( u \) such that \( \text{sgn}[u^{(N+1)}(t)] = (-1)^N \)

(ii) \( \tilde{x}_1 \) is an \( N \)th-degree increase in risk over \( \tilde{y}_1 \).

As an example of Corollary 1, if \( N = 3 \), so that we have \( \tilde{x}_1 \) exhibits more downside risk than \( \tilde{y}_1 \), as defined by Menezes et al. (1980), then precautionary saving will always be higher under \( \tilde{x}_1 \) if and only if preferences exhibit temperance.
4 Risky Interest Rates

We now assume that the interest paid on savings is itself random, rather than fixed. Even so-called "risk-free" bonds are typically only risk-free with respect to any default risk. There still might be some risk that the market rates will change, or that real purchasing power cannot be guaranteed due to unexpected inflation. Rothschild and Stiglitz (1971) considered a savings model with income only at date $t = 0$. At this date, the consumer decides how much to consume and how much to save, for consumption at date $t = 1$. Any amount saved earns a rate of interest $\tilde{r}$, where we assume $\tilde{r} > -1$. The consumer’s objective can thus be written as

$$\max_s U(s) \equiv u(y_0 - s) + \frac{1}{1 + \delta} E u(s \tilde{R}),$$  \hspace{1cm} (5)

where, for ease of notation, $\tilde{R}$ denotes the gross rate of interest, $\tilde{R} = (1 + \tilde{r})$.

The first-order condition for (5) is

$$U'(s) \equiv -u'(y_0 - s) + \frac{1}{1 + \delta} E[u'(s \tilde{R}) \tilde{R}] = 0.$$  \hspace{1cm} (6)

It is again straightforward to show that $U(s)$ is strictly concave, so that second-order conditions for a maximum easily hold and any solution to (6) is unique. We assume
that the optimal level of savings $s^*$ is strictly positive, with $s^* < y_0$.

Rothschild and Stiglitz (1971) consider a change in the distribution of $\tilde{R}$ to one that is a mean-preserving increase in risk, as defined by their earlier paper Rothschild and Stiglitz (1970). We consider a more general stochastic change in interest from say $\tilde{R}_a$ to $\tilde{R}_b$. We first consider the case where $\tilde{R}_a$ dominates $\tilde{R}_b$ by NSD.

**Proposition 2** Let $s_i$ denote the optimal level of savings for (5) when $\tilde{R} = \tilde{R}_i$, for $i = a, b$. The following two statements are equivalent:

(i) The optimal level of savings under $\tilde{R}_b$ is as least as high as under $\tilde{R}_a$, for every utility function $u$ such that $[-tu^{(n+1)}(t)/u^{(n)}] \geq n$ for all $n = 1, 2, ..., N$

(ii) $\tilde{R}_a$ dominates $\tilde{R}_b$ via NSD.

**Proof.** From (6) and the concavity of $U(s)$, we know that $s_b \geq s_a$ whenever

$$E[u'(s_a \tilde{R}_b)\tilde{R}_b] \geq E[u'(s_a \tilde{R}_a)\tilde{R}_a].$$

From NSD equivalence, this will hold for all $y_0$ whenever the function $h(R) \equiv Ru'(s_a R)$ satisfies the property that $\text{sgn}(h^{(n)}(R)) = (-1)^n$ for all $n = 1, 2, ..., N$. We proceed by induction. For $N = 1$, straightforward calculation shows that $h'(R) = s_a Ru''(s_a R) + u'(s_a R)$. Thus $h'(R) \leq 0$ holds for all $R$ if and only if relative risk aversion is greater than one: $-tu''(t)/u'(t) \geq 1, \forall t > 0$. 

---

7Since we assume $\overline{r} > -1$, the assumption of $s^* > 0$ avoids any issues associated with bankruptcy. Taken together with the assumption that $s^* < y_0$, we are simply assuming that the consumer must have positive consumption in each period.
For any \( n > 1 \), it follows from standard induction arguments that 
\[
    h^{(n)}(R) = (s_a)^n Ru^{(n+1)}(s_aR) + n(s_a)^{n-1}u^{(n)}(s_aR).
\]
Since by assumption \( s_a \) is strictly positive, it follows that 
\[
    h^{(n)}(R) \leq [\geq]0 \text{ for all } R \text{ if and only if } -tu^{(n+1)}(t)/u^{(n)}(t) \geq [\leq] n,
\]
\( \forall t > 0 \), so long as \( u^{(n)}(t) \neq 0 \). Applying NSD equivalence, it follows that \( \tilde{R}_a \)
dominates \( \tilde{R}_b \) via NSD is equivalent to 
\[
    E[u'(s_a\tilde{R}_b)\tilde{R}_b] \geq E[u'(s_a\tilde{R}_a)\tilde{R}_a]
\]
for all utility \( u \) such that 
\[
    -tu^{(n+1)}(t)/u^{(n)}(t) \geq n, \forall t > 0, \forall n = 1, 2, ..., N.
\]
The result then follows immediately.

If \( u^{(n+1)}(t) = 0 \) for all \( t > 0 \), then \( h^{(n)}(R) \) is a constant, possibly identical to zero. Consequently, any \( n^{th} \)-degree increase risk for interest rates will have no effect on the level of optimal level of savings. ■

It is noteworthy to compare the results from the two Propositions. Consider, for the sake of concreteness, the case where \( N = 3 \). In the case where the rate of interest was fixed, but future labor income was risky, we required only that the consumer be risk averse, prudent and temperate, in order for a third-order stochastic dominance deterioration in the distribution of future labor income to increase the level of precautionary savings. But in the case where we have only the interest rate being risky, there are in a sense two effects. One effect of the deterioration of the interest rate distribution is to save less as a type of substitution effect. Since the savings vehicle is now less attractive, the consumer will substitute some current
consumption for savings. This reduces her exposure to the riskier interest. On the other hand, the increased interest-rate risk makes future consumption riskier at date $t = 1$. This future uncertainty induces the consumer to save more by way of a precautionary demand for savings. The final effect on savings depends on the relative strength of this precautionary effect vis-à-vis the substitution effect. Since third-order stochastic dominance also allows for first-order and second-order dominance, the strength of this precautionary can only be guaranteed to be strong enough if (1) relative risk aversion exceeds one, (2) relative prudence exceeds two, and (3) relative temperance exceeds four.

For $N = 4$, we need to add $-tu^{(5)}(t)/u^{(4)}(t) \geq 5$, which using nomenclature from Lajeri-Chaherli (2004) we can describe as the measure of "relative edginess" exceeding five. For $n \geq 6$, we are unaware of any literature describing or naming the measure $-tu^{(n)}(t)/u^{(n-1)}(t)$. However, the general condition under expected utility that $\text{sgn}[u^{(n)}] = (-1)^{n+1}$ is discussed by Caballe and Pomansky (1995), who label $-u^{(n)}(t)/u^{(n-1)}(t)$ as a measure of "$n^{th}$-degree risk aversion." We thus might wish to label $-tu^{(n)}(t)/u^{(n-1)}(t)$ as a measure of "relative $n^{th}$-degree risk aversion."

In a manner similar to Corollary 1, we can induce from Proposition 2 the following result.

**Corollary 2:** Let $s_i$ denote the optimal level of savings for (5) when $\tilde{R} = \tilde{R}_i$, for
i = a, b. The following two statements are equivalent:

(i) The optimal level of savings under $\tilde{R}_b$ is as least as high as under $\tilde{R}_a$, for every utility function $u$ such that $[-tu^{(N+1)}(t)]/u^{(N)}(t) \geq N$, so long as $u^{(N)}(t) \neq 0 \ \forall t > 0$. For $u^{(N)}(t) = 0$, the savings levels will be identical.

(ii) $\tilde{R}_b$ is an $N^{th}$-degree increase in risk over $\tilde{R}_a$.

The result of Rothschild and Stiglitz (1971) is a special case of Corollary 2, with $N = 2$: If $\tilde{R}_b$ is mean-preserving increase in risk over $\tilde{R}_a$, then $s_b \geq s_a$ whenever relative prudence exceeds two.

It is interesting to note a few particular utility functions. Consider the case of quadratic utility, with $u(t) = t - kt^2$, where $k > 0$ and we restrict $t < (2k)^{-1}$. If we further restrict $t$ such that $(4k)^{-1} < t < (2k)^{-1}$, then relative risk aversion exceeds one, so that any first-degree increase in risk from $\tilde{R}_b$ to $\tilde{R}_a$ will increase savings. But an increase in $N^{th}$-degree risk for any $N$ other than $N = 1$, will lead to no change in savings.

For utility that belongs to the often-used class of functions exhibiting constant relative risk aversion (CRRA), with $-tu''(t)/u'(t) = \gamma \ \forall t > 0$, where $\gamma > 0$, it follows that $[-tu^{(N+1)}(t)]/u^{(N)}(t) = (\gamma - 1) + N$, which of course exceeds $N$ whenever $\gamma \geq 1$.

Thus, for CRRA utility, relative risk aversion larger than one is both necessary and

---

\[1 \quad 8\text{CRRA utility takes the form } u(t) = \ln t \text{ or } u(t) = \frac{1}{1-\gamma} t^{1-\gamma} \text{ for } \gamma \neq 1.\]
sufficient for any $N^{th}$-degree increase in risk to increase the level of savings.

5 Concluding Remarks

We examined two models of saving with a risky future. When future labor income is risky, a stochastic increase in $N^{th}$-degree risk leads to more savings if and only if $\text{sgn}[u^{(N+1)}] = (-1)^N$. When there is an increase in $N^{th}$-degree risk in the return on saving, this condition is no longer sufficient, and we require that the measure of relative $(N+1)^{st}$-degree risk aversion exceed $N$, i.e., $[\text{sgn}(u^{(N+1)}(t))]/u^{(N)}(t) \geq N$. This condition guarantees a precautionary effect that is strong enough to increase the level of savings.

References


