Valuing climate policies in an uncertain world
Take-home message

• Most short-term risks are affordable (EP puzzle, insurance puzzle). We should be approximately risk-neutral towards them.

• Many long-term risks are critical. Prudence and risk aversion matter a lot in their valuation and management.
  — Parametric uncertainty on growth, and on impacts
  — Fat tails, extreme events

• Intergenerational risk sharing and the pricing of life insurance products/liabilities.

• Climate change, nuclear industry, sustainability…. 
References

- IPCC AR 5 VOL3 CHAP3 (not chapter 2!)
Climate change: *Temperature hockeysticks*

Source: Chapman and Davis (2010)
Scientific ambiguity in climate change

Figure 1: Estimated probability density functions for the climate sensitivity from a variety of published studies, collated by Meinshausen et al. (2009).
Economics: A dismal science?

• There is a myriad of possible actions to make future generations better off.
  — education, freedom, democracy, equity,…
  — infrastructure: water, roads, internet,…
  — fighting climate change,…
• Most of these investments are costly. We must make choices and compromises.
• Compare costs and benefits. This cannot be criticized, except if this is badly performed.
• Many economists have been criticizing aggressive climate policies as not being the best thing to do for future generations. Are they right?
• Valuation of the future? *Intensity* of our efforts.
Past generations

What did they do for our future?
There is far more carbon in the ground than emitted in any baseline scenario.

Based on IPCC AR5 SRREN Figure 1.7
Our problem

What should we do for the future?
What should we do for the future?

• Ethical finance?
• Let’s examine the impact of our actions on the intergenerational welfare.
  — Time, risk, inequity, value, …
  — How do our actions affect IW?
• For safe projects, select a minimum rate of return (inclusive of all socio-economic benefits).
• Adjust this minimum rate of return for the riskiness of the projects.
Financial puzzles in the XXth century

• Do we do enough for the distant future?
  — Are interest rates low enough to induce us to invest for the future?
  — Western world in the XXth century: ~1%.
  — DEU/CCAPM models: ~4%.
  — Risk free rate puzzle: they invested too much!

• Do we take too much risk?
  — Is the equity risk premium (and the WACC) large enough to limit entrepreneurs’ risk taking?
  — Western world in the XXth century: ~5%.
  — DEU/CCAPM models: ~0.5%.
  — Equity premium puzzle: they didn’t take enough risk!
### Annualized real returns of equity and bonds from 1900 to 2006

<table>
<thead>
<tr>
<th>Country</th>
<th>Bill</th>
<th>Bond</th>
<th>Equity</th>
</tr>
</thead>
<tbody>
<tr>
<td>Australia</td>
<td>0.6%</td>
<td>1.3%</td>
<td>7.8%</td>
</tr>
<tr>
<td>Canada</td>
<td>1.6%</td>
<td>2.0%</td>
<td>6.3%</td>
</tr>
<tr>
<td>Denmark</td>
<td>2.3%</td>
<td>3.0%</td>
<td>5.4%</td>
</tr>
<tr>
<td>France</td>
<td>-2.9%</td>
<td>-0.3%</td>
<td>3.7%</td>
</tr>
<tr>
<td>Italy</td>
<td>-3.8%</td>
<td>-1.8%</td>
<td>2.6%</td>
</tr>
<tr>
<td>Japan</td>
<td>-2.0%</td>
<td>-1.3%</td>
<td>4.5%</td>
</tr>
<tr>
<td>Netherlands</td>
<td>0.7%</td>
<td>1.3%</td>
<td>5.4%</td>
</tr>
<tr>
<td>United Kingdom</td>
<td>1.0%</td>
<td>1.3%</td>
<td>5.6%</td>
</tr>
<tr>
<td>Sweden</td>
<td>1.9%</td>
<td>2.4%</td>
<td>7.9%</td>
</tr>
<tr>
<td>Switzerland</td>
<td>0.8%</td>
<td>2.1%</td>
<td>5.3%</td>
</tr>
<tr>
<td>USA</td>
<td>1.0%</td>
<td>1.9%</td>
<td>6.6%</td>
</tr>
</tbody>
</table>
Discounted marginal damage of one tCO2

<table>
<thead>
<tr>
<th></th>
<th>Discount rate</th>
<th>Social value of CO2</th>
</tr>
</thead>
<tbody>
<tr>
<td>Nordhaus</td>
<td>5%</td>
<td>8 $/tCO2</td>
</tr>
<tr>
<td>Stern/Hope</td>
<td>1.4%</td>
<td>85 $/tCO2</td>
</tr>
</tbody>
</table>
The normative approach to discounting

• Let us consider a marginal investment financed by a reduction of current consumption.
• In a growing economy, such an investment raises intergenerational inequalities.
• Under inequality aversion, the discount rate is the minimum rate of return necessary to compensate for this adverse effect of investment.
• Ramsey rule:
  \[ r_f = \text{inequality aversion} \times \text{prospective growth rate} = 2 \times 2\% = 4\% \]
Estimation of inequality aversion under the veil of ignorance

- You are indifferent between
  - 50-50 chance to live with a daily income of 80 or 120;
  - A sure daily income of X.

<table>
<thead>
<tr>
<th>$\gamma$</th>
<th>Certainty equiv $(80,1/2;120,1/2)$</th>
<th>Certainty equiv $(50,1/2;150,1/2)$</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>100,00</td>
<td>100,00</td>
</tr>
<tr>
<td>0.5</td>
<td>98,99</td>
<td>93,30</td>
</tr>
<tr>
<td>1</td>
<td>97,98</td>
<td>86,60</td>
</tr>
<tr>
<td>1.5</td>
<td>96,98</td>
<td>80,38</td>
</tr>
<tr>
<td>2</td>
<td>96,00</td>
<td>75,00</td>
</tr>
<tr>
<td>4</td>
<td>92,44</td>
<td>62,24</td>
</tr>
</tbody>
</table>
Other calibrations of the Ramsey rule

- Add “impatience” or “pure preference for the present”.

<table>
<thead>
<tr>
<th>Author</th>
<th>Inequality Aversion $\gamma$</th>
<th>Growth rate $g$</th>
<th>Discount Rate $\gamma/g$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Stern (1977)</td>
<td>2</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Cline (1992)</td>
<td>1.5</td>
<td>1%</td>
<td>1.5%</td>
</tr>
<tr>
<td>IPCC (1995)</td>
<td>1.5-2</td>
<td>1.6% - 8%</td>
<td>2.4% - 16%</td>
</tr>
<tr>
<td>Arrow (1999)</td>
<td>2</td>
<td>2%</td>
<td>4%</td>
</tr>
<tr>
<td>UK: Green Book (HM Treasury, 2003)</td>
<td>1</td>
<td>2%</td>
<td>2%</td>
</tr>
<tr>
<td>France: Rapport Lebêgue (2005)</td>
<td>2</td>
<td>2%</td>
<td>4%</td>
</tr>
<tr>
<td>Stern (2007)</td>
<td>1</td>
<td>1.3%</td>
<td>1.3%</td>
</tr>
<tr>
<td>Arrow (2007)</td>
<td>2-3</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Dasgupta (2007)</td>
<td>2-4</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Weitzman (2007a)</td>
<td>2</td>
<td>2%</td>
<td>4%</td>
</tr>
<tr>
<td>Nordhaus (2008)</td>
<td>2</td>
<td>2%</td>
<td>4%</td>
</tr>
<tr>
<td>Pindyck (2013)</td>
<td>1-3</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
Historical perspective....

- 2%: Myopia?

Source: R. Gordon, Northwestern U
Prudence: An argument to reduce the discount rate

- Precautionary motive to invest.

Source: R. Gordon, Northwestern U
How should uncertainty affect the discount rate?

- Precautionary motive to invest: Prudence.
- The uncertainty should reduce the discount rate.
- Can this solve the risk-free rate puzzle?
Efficient discount rates under uncertainty: CCAPM

- Intergenerational welfare function: \( W_0 = E \left[ \int_0^t e^{-\delta t} u(c_t) dt \right] \)

- We take the stochastic consumption path \( \{c_t : t \geq 0\} \) as given.

- Consider a marginal investment \( \{F_t : t \geq 0\} \). It raises the SWF iff

\[
\frac{\partial}{\partial \varepsilon} E \left[ \int_0^t e^{-\delta t} u(c_t + \varepsilon F_t) dt \right] \bigg|_{\varepsilon = 0} \geq 0
\]

\[
\int_0^t E \left[ \frac{e^{-\delta t} F_t u'(c_t)}{u'(c_0)} \right] dt \geq 0
\]

\[
\int_0^t e^{-\rho_t(F_t) t} EF_t dt \geq 0
\]

\[
\rho_t(F_t) = r_{ft} + \pi_t(F_t)
\]

\[
\begin{align*}
  r_{ft} &= \delta - \frac{1}{t} \ln \frac{E u'(c_t)}{u'(c_0)} \\
  \pi_t(F_t) &= -\frac{1}{t} \ln \frac{EF_t u'(c_t)}{EF_t u'(c_t)}
\end{align*}
\]
The extended Ramsey rule with a representative agent

\[ r = \delta - \ln \left( \frac{Eu'(c_1)}{u'(c_0)} \right) \]

- Sure growth rate \( g \):
  \[ c_1 = c_0 e^g \]
- Relative risk aversion:
  \[ R_1(c) = -c u''(c) / u'(c) \]
- DR:
  \[ r \approx \delta + R_1 g \]

<table>
<thead>
<tr>
<th>( \delta )</th>
<th>( g )</th>
<th>( R_1 )</th>
</tr>
</thead>
<tbody>
<tr>
<td>0%</td>
<td>2%</td>
<td>2</td>
</tr>
</tbody>
</table>

\[ r \approx 4\% \]

- Relative prudence:
  \[ R_2(c) = -c u'''(c) / u''(c) \]
- DR:
  \[ r \approx \delta + R_1 \left[ g - 0.5R_2 \sigma^2 \right] \]

<table>
<thead>
<tr>
<th>( \delta )</th>
<th>( g )</th>
<th>( \sigma )</th>
<th>( R_1 )</th>
<th>( R_2 )</th>
</tr>
</thead>
<tbody>
<tr>
<td>0%</td>
<td>2%</td>
<td>3.6%</td>
<td>2</td>
<td>3</td>
</tr>
</tbody>
</table>

\[ r = 3.61\% \]

Exact if CRRA
Calibration of the Ramsey rule around the world

<table>
<thead>
<tr>
<th>Country</th>
<th>g (wealth effect)</th>
<th>σ (precautionary effect)</th>
<th>Discount rate</th>
</tr>
</thead>
<tbody>
<tr>
<td>United States</td>
<td>1.74% (3.43%)</td>
<td>2.11% (-0.13%)</td>
<td>3.35%</td>
</tr>
<tr>
<td>France</td>
<td>1.75% (3.50%)</td>
<td>1.57% (-0.07%)</td>
<td>3.43%</td>
</tr>
<tr>
<td>Canada</td>
<td>1.79% (3.59%)</td>
<td>2.12% (-0.13%)</td>
<td>3.45%</td>
</tr>
<tr>
<td>United Kingdom</td>
<td>1.86% (3.71%)</td>
<td>2.18% (-0.14%)</td>
<td>3.57%</td>
</tr>
<tr>
<td>Japan</td>
<td>2.34% (4.07%)</td>
<td>2.61% (-0.20%)</td>
<td>4.47%</td>
</tr>
<tr>
<td>China</td>
<td>7.60% (15.20%)</td>
<td>3.53% (-0.37%)</td>
<td>14.82%</td>
</tr>
<tr>
<td>South Korea</td>
<td>5.38% (10.73%)</td>
<td>3.40% (-0.33%)</td>
<td>10.41%</td>
</tr>
<tr>
<td>Taiwan</td>
<td>5.41% (10.82%)</td>
<td>5.29% (-0.84%)</td>
<td>9.98%</td>
</tr>
<tr>
<td>India</td>
<td>3.34% (6.88%)</td>
<td>3.03% (-0.28%)</td>
<td>6.61%</td>
</tr>
<tr>
<td>Russia</td>
<td>1.54% (3.08%)</td>
<td>5.59% (-0.94%)</td>
<td>2.14%</td>
</tr>
<tr>
<td>Gabon</td>
<td>1.29% (2.58%)</td>
<td>9.63% (-2.78%)</td>
<td>-0.20%</td>
</tr>
<tr>
<td>Liberia</td>
<td>-1.90% (-3.79%)</td>
<td>19.58% (-11.50%)</td>
<td>-15.30%</td>
</tr>
<tr>
<td>Zaire (RDC)</td>
<td>-2.76% (-5.33%)</td>
<td>5.31% (-0.83%)</td>
<td>-6.38%</td>
</tr>
<tr>
<td>Zambia</td>
<td>-0.69% (-1.38%)</td>
<td>4.01% (-0.48%)</td>
<td>-1.86%</td>
</tr>
<tr>
<td>Zimbabwe</td>
<td>-0.26% (-0.53%)</td>
<td>6.50% (-1.27%)</td>
<td>-1.79%</td>
</tr>
</tbody>
</table>

Table 1: Country-specific discount rate computed from the extended Ramsey rule using the historical mean $g$ and standard deviation $\sigma$ of growth rates of real GDP/cap 1969-2010.
Much more uncertainty from cross-sectional growth data 1969-2011

Time series U.S.
\[ \mu = 42 \times 1.8\% \]
\[ \sigma = \sqrt{42} \times 3.6\% \]

Cross-section 190 countries 1969-2011
\[ \mu = 70.58\% \]
\[ \sigma = 71.42\% \]
The term structure of the safe discount rate

Should we do more for the more distant future?
Term structure

- CCAPM is usually calibrated for short-lived assets.
- Could it be desirable to use a 4% discount rate for short-lived safe assets, and a discount rate of 1% for long-lived safe assets?
  - Term structure of discount/interest rates.
  - Weitzman/Stern/Nordhaus on climate change.
- Attitude towards the accumulation of risks over time?
Term structure of the safe discount rate

- CCAPM with CRRA + random walk => If 3% is efficient for a 1-year maturity, it is also efficient for a 200-year maturity.
- The positive serial correlation in the growth rate of consumption magnifies the long term risk. Under prudence,
  - it reduces the long discount rate;
  - it justifies a decreasing term structure.
- Gollier (2012): Argument can be applied to all stochastic processes that exhibit positive concordance: mean-reversion, Markov regime switches, ....
Term structure of safe discount rates

\[ r_{ft} = \delta - \frac{1}{t} \ln \frac{Eu'(c_t)}{u'(c_0)} \]

\[ r_{f2} = \delta - \frac{1}{t} \ln \frac{Eu'(c_0 \exp(g_1 + g_2))}{u'(c_0)} \]

• What are the conditions under which \( Ef(Z, Y) \) is increased by the increased concordance/correlation between \( Z \) and \( Y \)?

• \( f \) is supermodular iff

\[ f(x, y) + f(\bar{x}, y) \geq f(x, \bar{y}) + f(\bar{x}, \bar{y}) \quad \forall x \leq \bar{x} \quad \forall y \leq \bar{y} \]

\[ f_{xy} \geq 0 \]
More convergence
Positive Quadrant Dependence

• Let $F_1$ and $F_2$ denote two cdf of $(z,y)$.
• We say that $F_2$ exhibits more convergence than $F_1$ if $F_2$ can be obtained from $F_1$ by a sequence of transfers of probability masses of the following nature:

$$F_2(z, y) \leq F_1(z, y) \quad \forall (z, y)$$
Generic result

• Any increase in concordance in serial growth rates reduces the long discount rate if relative prudence is larger than unity.
Valuation formulas using Cumulant-Generating Functions (CGF)

- We apply this under the standard CCAPM specification:
  \[ u'(c) = c^{-\gamma} \quad \text{and} \quad F_t = \xi_t c_t^\beta \]

- Define \( G_t = \ln c_t / c_0 \) and \( \chi(a,x) = \ln E \exp(ax) \).

- Risk free rate:
  \[ r_{ft} = \delta - \frac{1}{t} \ln E \left( \frac{c_t}{c_0} \right)^{-\gamma} = \delta - \frac{1}{t} \ln E \exp(-\gamma \ln \frac{c_t}{c_0}) = \delta - \frac{1}{t} \ln E \exp(-\gamma G_t) \]
  \[ r_{ft} = \delta - t^{-1} \chi(-\gamma, G_t) \]

- Similarly for the risk premium:
  \[ \pi_t(\beta) = t^{-1} \left( \chi(\beta, G_t) + \chi(-\gamma, G_t) - \chi(\beta - \gamma, G_t) \right) \]
Properties of CGF

Lemma 1: If it exists, the CGF function \( \chi(a,x) = \ln E \exp(ax) \) has the following properties:

i. \[ \chi(a,x) = \sum_{n=1}^{\infty} \kappa_n^x a^n / n! \text{ where } \kappa_n^x \text{ is the nth cumulant of random variable } x. \text{ If } m_n^x \text{ denotes the centered moment of } x, \text{ we have that } \kappa_1^x = Ex, \kappa_2^x = m_2^x, \kappa_3^x = m_3^x, \kappa_4^x = m_4^x - 3(m_2^x)^2, \ldots \]

ii. The most well-known special case is when \( x \) is \( N(\mu, \sigma^2) \), so that \( \chi(a,x) = a \mu + 0.5 a^2 \sigma^2 \).

iii. \( \chi(a,x + y) = \chi(a,x) + \chi(a,y) \) when \( x \) and \( y \) are independent random variables.

iv. \( \chi(0,x) = 0 \) and \( \chi(a,x) \) is infinitely differentiable and convex in \( a \).

v. \( a^{-1} \chi(a,x) \) is increasing in \( a \), from \( Ex \) to the supremum of the support of \( x \) when \( a \) goes from zero to infinity.

vi. The cumulant of the nth order is homogeneous of degree \( n \): \( \kappa_n^{ax} = \lambda^n \kappa_n^x \) for all \( \lambda \in \mathbb{R} \).
Gaussian case

- Suppose that \( G_t = \ln \frac{c_t}{c_0} \sim N(\mu t, \sigma^2 t) \).

\[
\begin{align*}
    r_{ft} &= \delta - t^{-1} \chi(-\gamma, G_t) \\
    \pi_t(\beta) &= t^{-1} \left( \chi(\beta, G_t) + \chi(-\gamma, G_t) - \chi(\beta - \gamma, G_t) \right) \\
    &\Downarrow \\
    r_{ft} &= \delta + \gamma \mu - 0.5 \gamma^2 \sigma^2 = r_f \\
    \pi_t(\beta) &= \beta \pi \quad \text{with} \quad \pi = \gamma \sigma^2
\end{align*}
\]

- Observations:
  - Wealth effect and precautionary effect;
  - Constant term structures;
  - The risk premium is proportional to the beta.
The term structure of the safe discount rate

The case of ambiguity in the distribution of annual growth rate
Ambiguity aversion in the EU model!

• Halevy and Feltkamp (REStud, 2005).

\[ X_1 \sim (A, \theta; B, 1-\theta) \]
\[ \theta \sim (0.1, 1/2; 0.9, 1/2) \]
\[ Y \sim (A, 1/2; B, 1/2) \]

\[ X_1, X_2 \text{ i.i.d.} \sim (A, \theta; B, 1-\theta) \]
\[ \theta \sim (0.1, 1/2; 0.9, 1/2) \]
\[ X_1 + X_2 \sim (2A, 41\%; A + B, 18\%; 2B, 41\%) \]

\[ X_1, X_2 \text{ i.i.d.} \sim (A, \theta; B, 1-\theta) \]
\[ \theta = 0.5 \]
\[ X_1 + X_2 \sim (2A, 25\%; A + B, 50\%; 2B, 25\%) \]
Intuition: fat tails \( \mu \sim (1\%, 1/2; 3\%, 1/2) \)
\( \sigma = 3.6\% \)
Ambiguous random walk

• Suppose that

\[ g_t = \ln c_{t+1} / c_t \sim \text{i.i.d.} \]
\[ g \sim CDF_\theta \]
\[ \theta \sim H \]

• Then, we have

\[ \chi(k, G_t) = \ln E_\theta \left[ E \left[ \exp \left( k \sum_{\tau=0}^{t-1} g_\tau \right) \bigg| \theta \right] \right] \]
\[ = \ln E_\theta \left[ \left( E \left[ \exp (kg) \bigg| \theta \right] \right)^t \right] \quad \text{(i.i.d.)} \]
\[ = \ln E_\theta \left[ \exp t \ln \left( E \left[ \exp (kg) \bigg| \theta \right] \right) \right] \]
\[ = \ln E_\theta \left[ \exp t \chi(k, g \big| \theta) \right] = \chi(t, \chi(k, g \big| \theta)) \]
Properties of the term structure of risk-free discount rates

\[ r_{ft} = \delta - t^{-1} \chi(-\gamma, G_t) = \delta - t^{-1} \chi(t, \chi(-\gamma, g|\theta)) \]

- The term structure of the risk-free rate is decreasing.
- For small maturities, parametric uncertainty does not matter:
  \[
  \lim_{t \to 0} r_{ft} = \lim_{t \to 0} \left( \delta - t^{-1} \chi(t, \chi(-\gamma, g|\theta)) \right)
  = \delta - E\chi(-\gamma, g|\theta)
  = \delta - \left[ \sum_{n=1}^{\infty} \frac{(-\gamma)^n}{n!} E[k_n^{g|\theta}] \right]
  
  \]
- For long maturities, it tends to the smallest possible discount rate:
  \[
  \lim_{t \to \infty} r_{ft} = \lim_{t \to 0} \left( \delta - t^{-1} \chi(t, \chi(-\gamma, g|\theta)) \right)
  = \delta - \sup_\theta \chi(-\gamma, g|\theta)
  = \delta - \sup_\theta \left\{ \sum_{n=1}^{\infty} \frac{(-\gamma)^n}{n!} k_n^{g|\theta} \right\}
  \]
How fast does the risk-free DR decrease with maturities?

- Property $i$: $\chi(t, x) = \sum_{n=1}^{\infty} \frac{\kappa_n x^n}{n!} \Rightarrow \lim_{t \to 0} \frac{\partial}{\partial t} \left( \frac{1}{t} \chi(t, x) \right) = \frac{\text{Var}(x)}{2!}$

- This implies that

$$\lim_{t \to 0} \frac{\partial r_{ft}}{\partial t} = \lim_{t \to 0} \frac{\partial}{\partial t} \left( \delta - t^{-1} \chi(t, \chi(-\gamma, g | \theta)) \right)$$

$$= -0.5 \text{Var} \left( \chi(-\gamma, g | \theta) \right)$$

- Suppose that the uncertainty about the distribution of $g$ is concentrated in its $n$th cumulant.

$$\lim_{t \to 0} \frac{\partial r_{ft}}{\partial t} = -\frac{\gamma^{2n}}{2(n!)^2} \text{Var} \left( \kappa_n^{g|\theta} \right)$$
Preliminary conclusion

• Parametric uncertainty on the random walk governing consumption growth magnifies the uncertainty affecting distant consumption.
• Under prudence, this should bias our investment decisions towards safe projects with distant benefits.
• This must be done by using a decreasing term structure of discount rates.
• This is good for economists concerned by sustainability!
The term structure of the risk premium
Term structure of the risk premium?

- Exactly the same argument applies for an increasing term structure of the risk premium (RP), under risk aversion:
  - Flat term structure of the RP under a random walk;
  - Persistence of shocks magnifies long-term risk;
  - It justifies an increasing term structure of the RP.

- If the climate beta is large enough, the DR that should be used to discount expected climate benefits is *increasing*!

- Recent developments in finance theory: Long Run Risk: Bansal et al., Hansen et al., …
Evaluation of public investments in France

  — Risk free rate: 2.5% (less than 50 years) to 1.5% (LT).
  — Risk premium: 1.5% (less than 50 years) to 3% (LT).

• International comparisons:
  — US: 7%;
  — Norway, UK: 4% (ST) and 2% (LT).
Three Double Chi operations for the risk premium

\[ \pi_i(\beta) = t^{-1} \chi(t, \chi(\beta, g|\theta)) + t^{-1} \chi(t, \chi(-\gamma, g|\theta)) - t^{-1} \chi(t, \chi(\beta-\gamma, g|\theta)) \]
Properties of the term structure of risk premia

\[ \pi_t(\beta) = t^{-1} \chi(t, \chi(\beta, g|\theta)) + t^{-1} \chi(t, \chi(-\gamma, g|\theta)) - t^{-1} \chi(t, \chi(\beta - \gamma, g|\theta)) \]

- For small maturities, parametric uncertainty does not matter.

\[ \lim_{t \to 0} \pi_t(\beta) = E\left[ \chi(\beta, g|\theta) + \chi(-\gamma, g|\theta) - \chi(\beta - \gamma, g|\theta) \right] \]

\[ = \sum_{n=1}^{\infty} \frac{\beta^n + (-\gamma)^n - (\beta - \gamma)^n}{n!} E\left[ \kappa_n^{g|\theta} \right] \]

- For large maturities, it tends to

\[ \lim_{t \to \infty} \pi_t(\beta) = \sup \chi(\beta, g|\theta) + \sup \chi(-\gamma, g|\theta) - \sup \chi(\beta - \gamma, g|\theta) \]

- The risk premium is generally not proportional to \( \beta \).
Slope of the term structure of risk premia

- If we suppose that the uncertainty about the distribution of \( g \) is concentrated in its \( n \)th cumulant, we have that
  
  \[
  \pi_t(\beta) = t^{-1} \left( \chi(t, \chi(\beta, g|\theta)) + \chi(t, \chi(-\gamma, g|\theta)) - \chi(t, \chi(\beta-\gamma, g|\theta)) \right)
  \]

  \[
  \downarrow
  
  \lim_{t \to 0} \frac{\partial \pi_t(\beta)}{\partial t} = \frac{\beta^{2n} + \gamma^{2n} - (\beta - \gamma)^{2n}}{2(n!)^2} \text{Var}\left( \kappa_n^{g|\theta} \right)
  \]

- The ratio in the RHS has the same sign as \( \beta \).
- The aggregate risk premium is increasing for short maturities.
The term structure of the risk-adjusted discount rate

\[ \frac{\partial \rho_t(\beta)}{\partial t} \bigg|_{t=0} = \frac{\beta^{2n} - (\beta - \gamma)^{2n}}{2(n!)^2} \text{Var}(\kappa_{n}^{g|\rho}) \]

• Proposition: Suppose that only one cumulant of \( g \) is uncertain. The rate at which one should discount a project \( \beta \) is increasing with maturity for small \( t \) if and only if \( \beta > \gamma/2 \).
Main results

• I assumed here that the growth process is i.i.d., but with parametric uncertainty. It implies

\[
    r_{ft} = \delta - t^{-1} \chi(t, \chi(-\gamma, g | \theta))
\]

\[
    \pi_t(\beta) = t^{-1} \chi\left(t, \chi\left(\beta, g | \theta\right)\right) + t^{-1} \chi\left(t, \chi\left(-\gamma, g | \theta\right)\right) - t^{-1} \chi\left(t, \chi\left(\beta - \gamma, g | \theta\right)\right)
\]

• Parametric uncertainty implies
  — No effect on the short-term risk-free rate and risk premium;
  — A decreasing term structure of the risk free rate;
  — An increasing term structure of the risk premium;
  — A decreasing (increasing) term structure of the discount rate if the asset’s beta is smaller (larger) than half the risk aversion;
  — The asset’s risk premium is not proportional to the asset’s beta.
APPLICATION 1

Unknown trend or volatility of the growth rate of consumption
Uncertain volatility

• Suppose that $c_t$ follow a geometric Brownian motion with an unknown volatility.

$$r_{ft} = \delta + \gamma \mu - 0.5 \gamma^2 E\left[\sigma^2 | \theta \right] - \frac{1}{8} \gamma^4 t Var(\sigma^2 | \theta) - \frac{1}{48} \gamma^6 xt^2 Skew(\sigma^2 | \theta) - ...$$

$$\pi_t(\beta) = \beta \gamma E\left[\sigma^2 | \theta \right] + \frac{1}{2} \beta \gamma Var(\sigma^2 | \theta) \left[ \beta^2 - \frac{3}{2} \beta \gamma + \gamma^2 \right] t + ...$$

• Weitzman (2007) considers the extreme case of an inverted gamma distribution for the variance of $g$. It implies a Student-t distribution for $g$.

• The inverted gamma distribution has no real-valued CGF. It implies that

$$r_{ft} = -\infty \text{ and } \pi_t(1) = +\infty.$$
Uncertain trend

- If the trend is normally distributed, then
  - the term structure of the DR is linearly decreasing in $t$.
  - the term structure of the risk premium is linearly increasing in $t$.

- For distributions with a bounded support:

$$
\rho_\infty(\beta) = \begin{cases} 
\delta + \gamma \sigma^2 (\beta - 0.5 \gamma) + \gamma \mu_{\min} & \text{if } \beta \leq 0 \\
\delta + \gamma \sigma^2 (\beta - 0.5 \gamma) + (\gamma - \beta) \mu_{\min} + \beta \mu_{\max} & \text{if } 0 < \beta \leq \gamma \\
\delta + \gamma \sigma^2 (\beta - 0.5 \gamma) + \gamma \mu_{\max} & \text{if } \beta > \gamma 
\end{cases}
$$
Calibration with an unknown trend

\[ g | \mu \sim N(\mu, \sigma^2) \]
\[ \mu \sim U[0\%, 3\%] \]
\[ \delta = 0 \]
\[ \gamma = 2 \]
\[ \sigma = 4\% \]
Extension to mean-reversion

- Long-term risk à la Bansal and Yaron (2004):

\[
\ln c_{t+1} / c_t = x_t
\]

\[
x_t = \mu + y_t + \epsilon_{xt},
\]

\[
y_t = \phi y_{t-1} + \epsilon_{yt},
\]

\[
\rho_t(\beta) = \delta + \gamma y_{t-1} \phi \frac{1 - \phi'}{t(1 - \phi)} + \gamma (\beta - 0.5\gamma) \left( \frac{\sigma_y^2}{(1 - \phi)^2} \left[ 1 - 2\phi \frac{\phi' - 1}{t(\phi - 1)} + \phi^2 \frac{\phi^2 - 1}{t(\phi^2 - 1)} \right] + \sigma_x^2 \right) + t^{-1} (\chi(t, \beta \mu) - \chi(t, (\beta - \gamma) \mu))
\]

- Calibration based on Bansal and Yaron (2004) (monthly)

\[
\delta = 0, \gamma = 2, \sigma_x = 0.78\%, \sigma_y = 0.034\%, \phi = 0.979, \mu \sim (0.05\%, 1/2; 0.25, 1/2).
\]
Term structures as a function of short-term expectations
APPLICATION 2

Unknown probability of macroeconomic catastrophe
Barro’s model of macro catastrophes

- Assumption on the distribution of growth:

\[ g \sim (h_1, 1-p; h_2, p) \quad \text{with} \quad h_i \sim N(\mu_i, \sigma_i^2) \]

- Barro (2006, 2009) and Martin (2012) calibrate the CCAPM model with

\[ \delta = 3\% \]
\[ \gamma = 4 \]
\[ \mu_1 = 2.5\% \quad \sigma_1 = 2\% \]
\[ \mu_2 = -39\% \quad \sigma_2 = 25\% \]

- Barro (2006) also assumes \( p = 1.7\% \):

  - 60 catastrophes in 35 countries over the last 100 years: \( p = 60/3500 \).

- The term structures are flat with \( r_f = 0.5\% \)

\[ \pi(1) = 5.9\% \]
Uncertain frequency of catastrophes

• Because $p$ is small, it is highly difficult to estimate with precision. Moreover, the interest rate and the risk premium are highly sensitive to $p$.

• Bayesian approach with diffuse prior: posterior $p \sim Beta(61,3441)$. 

![Graph showing the posterior distribution with $Ep = 1.7\%$]
Term structures with catastrophes of uncertain frequency

• No effect of the uncertain frequency on short term discount rates:
  \[ \lim_{t \to 0} r_{ft} = 0.5\% \]
  \[ \lim_{t \to 0} \pi_t(1) = 5.9\% \]

• But the asymptotic discount rates are very different:
  \[ \lim_{t \to \infty} r_{ft} = -203\% ! \]
  \[ \lim_{t \to \infty} \pi_t(1) = 63\% ! \]

• These asymptotic values are independent of the quality of the information!
### Term structures in the Barro-Bayes model

<table>
<thead>
<tr>
<th>maturity</th>
<th>t=1</th>
<th>t=10</th>
<th>t=100</th>
<th>t=250</th>
</tr>
</thead>
<tbody>
<tr>
<td>Risk free rate</td>
<td>0.2%</td>
<td>0.1%</td>
<td>-1.0%</td>
<td>-3.5%</td>
</tr>
<tr>
<td>Risk premium ($\beta=1$)</td>
<td>6.0%</td>
<td>6.1%</td>
<td>7.0%</td>
<td>8.9%</td>
</tr>
<tr>
<td>Risk premium ($\beta=2$)</td>
<td>9.2%</td>
<td>9.3%</td>
<td>10.4%</td>
<td>12.7%</td>
</tr>
</tbody>
</table>
What is the climate beta?
The climate beta

• Does mitigation have an insurance value?
• The negative-beta theory:
  — A high climate sensitivity implies a low GDP and a high payoff from mitigation.
• The positive-beta theory:
  — A higher growth implies higher concentration of CO2, a larger marginal damage, and a larger payoff from mitigation.
  — This means that the climate beta is positive. “Those states in which the global temperature increase is particularly high are also ones in which we are on average richer in the future.” (Nordhaus 2011)
• My own estimation from DICE: $\beta = 1.32$. 
A two-period DICE model

- Prototype DICE model:
  \[ T = \omega_1 E \]  
  \[ E = \omega_2 Y - I_0 \]  
  \[ D = \theta_1 T^{\theta_2} \]  
  \[ Q = e^{-D} Y \]  
  \[ C = \alpha Q \]  

- Investment \( I_0 \) to reduce emissions.
- Two sources of uncertainty: climate sensitivity and economic growth.
Calibration

<table>
<thead>
<tr>
<th>Variable</th>
<th>Value</th>
<th>Remark</th>
</tr>
</thead>
<tbody>
<tr>
<td>$t$</td>
<td>50 years</td>
<td>Time horizon between dates 0 and 1.</td>
</tr>
<tr>
<td>$Y = e^{\sum_{i=1}^{\infty} x_i}$</td>
<td>$x_i \text{ iid } \sim N(\mu, \sigma^2)$</td>
<td>$Y_0$ is normalized to unity. The growth rate of production follows a normal random walk.</td>
</tr>
<tr>
<td>$\mu$</td>
<td>1.5%</td>
<td></td>
</tr>
<tr>
<td>$\sigma$</td>
<td>4%</td>
<td></td>
</tr>
<tr>
<td>$\omega_2$</td>
<td>1</td>
<td>Normalization</td>
</tr>
<tr>
<td>$\omega_1$</td>
<td>0.45</td>
<td>This implies that the expected increase in temperature in the next 50 years equals $\xi EY = 1^\circ C$.</td>
</tr>
<tr>
<td>$\theta_2$</td>
<td>1.5</td>
<td>Center of the “consensus interval” [1,2].</td>
</tr>
<tr>
<td>$\theta_1$</td>
<td>$\sim U[0%, 5%]$</td>
<td>This means that the damage at the average temperature increase of $1^\circ C$ is uniformly distributed on $[0%, 5%]$ of pre-damage production.</td>
</tr>
<tr>
<td>$\alpha$</td>
<td>0.75</td>
<td>Consumption equals 75% of post-damage production.</td>
</tr>
</tbody>
</table>
Benefits from mitigation are procyclical

Benefits from mitigation in 2064

\[ \ln F = 1.3 \ln C + \varepsilon \]
Fighting climate change along the business cycle

Early in a recession, one should give priority to investments yielding immediate reliefs.

At the same time, one should maintain a high cost of carbon.
Conclusion

• The parametric uncertainty affecting the distribution of the iid growth rate of the economy justifies using
  — a decreasing term structure of the risk-free discount rate;
  — an increasing term structure of the aggregate risk premium.
  — a non-proportional risk premium formula.
• The risky discount rate is increasing with maturity if the beta is larger than half the relative risk aversion.
• The climate beta is larger than unity, which implies an increasing term structure under the standard CRRA=2.
Food for thought

The gamma discounting argument
Motivation

• Should one use a smaller rate to discount more distant benefits?
• Simple answer by Weitzman (1998, 2001): Yes, because future interest rates are uncertain.
• 1400 cites on Google Scholar…
• Norway, UK, France have used the argument for public policy evaluation.
Consider a simple safe project with
- an initial cost $C$,
- a future benefit $F$ occurring at date $t$.

If the interest rate is $r$, one can transfer the future benefit to today by a loan of $F \exp(-rt)$, yielding a net benefit

$$NPV = -C + F \exp(-rt)$$

Invest if NPV is positive.

Suppose now that $r$ is uncertain. A risk-neutral DM should invest if the expected NPV is positive, i.e., if the NPV is positive using a certainty equivalent DR $r_{0\rightarrow t}^W$ defined by

$$\exp(-r_{0\rightarrow t}^W t) = E \exp(-rt)$$
Gamma Discounting: Example

\[ r_{0 \rightarrow t}^W = -t^{-1} \chi(t, -r) \]

- \( r_{0 \rightarrow t}^W \) is decreasing and tends to the smallest possible \( r \).
- Weitzman (2001) proposes a gamma distribution for \( r \), hence the terminology.

\[ r \sim (1\%, 1/2; 6\%, 1/2) \]
But what if…

- If the interest rate is $r$, one can transfer the current cost to the terminal date $t$ by a loan of $C$, yielding a net benefit

$$NFV = -C \exp(rt) + F$$

- Invest if NFV is positive.
- Gollier (2004): Suppose now that $r$ is uncertain. A risk-neutral DM should invest if the expected NFV is positive, i.e., if the NPV is positive using a certainty equivalent DR $r^G_{0 \rightarrow t}$ defined by

$$\exp(r^G_{0 \rightarrow t} t) = E \exp(rt)$$
Expected NFV: Example

\[ r^G_{0 \rightarrow t} = t^{-1} \chi(t, r) \]

- \( r^G_{0 \rightarrow t} \) is increasing and tends to the largest possible \( r \).

\[ r \sim (1\%, 1/2; 6\%, 1/2) \]
The « Weitzman-Gollier » puzzle

• Lack of an economic foundation for gamma discounting, and for DDR.
• Under risk neutrality, interest rates are constant (and equal to the rate of impatience). We must add risk aversion into the picture.
Equivalence results

- **Proposition 1**: If the representative agent is a discounted expected utility maximizer, there are three equivalent ways to define the efficient long discount rate:

\[
e^{-2r_{0\to2}} = \frac{Eu'(c_2)}{u'(c_0)}
\]

\[
= \left( E \left[ \frac{u'(c_2)}{Eu'(c_2)} e^{r_{0\to1} + r_{1\to2}} \right] \right)^{-1}
\]

\[
= E \left[ \frac{u'(c_1)}{Eu'(c_1)} e^{-r_{0\to1} - r_{1\to2}} \right].
\]

- These characterizations fail to attain the objective envisioned by Weitzman to fully characterize the price of long-dated safe assets from the distribution of future spot interest rates alone.